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a)

$$\Delta G = -\frac{4}{3}\pi r^3 \Delta G_v + 4\pi r^2 \gamma \quad \mathrm{I}$$

II $nv = \frac{4}{3}\pi r^3 \to r = \left(\frac{3}{4\pi} n v\right)^{1/3}$ (nv = volume)

↓

plugging II in I

$$\Delta G = -nv\Delta G_v + \underbrace{4\pi\left(\frac{3}{4\pi}\right)^{2/3} n^{2/3} v^{2/3}\gamma}_{\mathrm{III}}$$

simplifying III

$$(4\pi)^{1/3} \times 9^{1/3} \times n^{2/3} \times v^{2/3} \times \gamma = (36\pi)^{1/3} \times n^{2/3} \times v^{2/3} \times \gamma \checkmark$$

also $v\Delta G_v = \Delta G_a v$

b) $\frac{\partial \Delta G}{\partial n} = 0 \to$

$$-v\Delta G_v + (36\pi)^{1/3} v^{2/3} \gamma \times \frac{2}{3} n^{-1/3} = 0$$

$$\to n^* = \left(\frac{(36\pi)^{1/3} v^{2/3}\gamma}{v\Delta G_v} \times \frac{2}{3}\right)^3 = \frac{36\pi \cdot \gamma^3}{v\Delta G_v^3} \times \frac{8}{27}$$

$$= \frac{32}{3} \frac{\pi \gamma^3}{\frac{(\Delta G_a)^3}{v^2}} = \frac{32}{3} \frac{\pi v^2 \gamma^3}{(\Delta G_a)^3}$$

for $\Delta G^* \rightarrow$ by plugging n^* in ΔG equation we will have $\Delta G^* = \frac{16}{3} \pi \frac{\gamma^3}{(\Delta G_v)^2}$

c) $\Delta G_{gra} = \Delta G_{di}$

$$\Delta G_{gr} = -n(G_v^\circ - G_{gra}^\circ) + n^{2/3} v_{gr}^{2/3} \gamma_{gr} (36\pi)^{1/3}$$

$$\Delta G_{dia} = -n(G_v^\circ - G_{dia}^\circ) + n^{2/3} v_{dia}^{2/3} \gamma_{dia} (36\pi)^{1/3}$$

$$\cancel{-nG_v^\circ} + nG_{gr}^\circ + n^{2/3} v_{gr}^{2/3} \gamma\, 36\pi^{1/3} = \cancel{-nG_v} + nG_{dia}^\circ + n^{2/3} v_{dia}^{2/3} \gamma_{dia}\, 36\pi^{1/3}$$

$$-n(G_{dia}^\circ - G_{gr}^\circ) = n^{2/3} (v_{dia}^{2/3} \gamma_{dia} - v_{gra}^{2/3} \gamma_{gr})(36\pi)^{1/3}$$

$$n = \frac{(v_{gr}^{2/3} \gamma_{gr} - v_{dia}^{2/3} \gamma_{dia})^3 36\pi}{(G_{dia}^\circ - G_{gra}^\circ)^3}$$

now by plugging values in the equation

for $\gamma_{dia} = 3.6\ J/m^2 \rightarrow$ 466 atom in a cluster

for $\gamma_{dia} = 3.65\ \frac{J}{m^2} \rightsquigarrow$ 146 atom in a cluster

for $\gamma_{dia} = 3.7\ \frac{J}{m^2} \rightsquigarrow$ 21 atom in a cluster

d) $\Delta G_{dia} < \Delta G_{gra} \rightsquigarrow$ diamond is energicaly favoured

So based on above calculations

$$n < \frac{\left(v_{gra}^{2/3}\gamma_{gra} - v_{dia}^{2/3}\gamma_{dia}\right)^3}{\left(G^{\circ}_{dia} - G^{\circ}_{gra}\right)^3} 36\pi$$ which means

each value of n calcualated in part c are critical value and n should be lower. In other words:

for $\gamma_{dia} = 3.6 \rightsquigarrow n < 466$ for $\gamma_{dia} = 3.65$ $n < 146$

for $\gamma_{dia} = 3.7 \rightsquigarrow n < 21$

e) $n^* = \frac{32}{3} \frac{\pi \gamma^3}{v \Delta G_v^3} \rightsquigarrow$ now by assuming $n^* = 100$

$\gamma_{gra} = 3.1 \, J/m^2$ & $v = 8 Å \Longrightarrow \Delta G_v = 1.077 \times 10^{10} \, J/m^3$

f) $\frac{I_{gra}}{I_{dia}} = \exp\left[\frac{\Delta G^*_{dia} - \Delta G^*_{gr}}{R.T}\right]$

and $\Delta G^*_{gra} = 4.28 \times 10^{-18} \, J$

only ΔG^*_{dia} is changing in each time

so by calculating ΔG^* with respect to γ_{dia}

we will have

for $\gamma_{dia} = 3.6 \, J/m^2 - \Delta G^*_{dia} = 4.06 \times 10^{-18} \longrightarrow I_{ratio} = 8.7 \times 10^{-24}$

for $\gamma_{dia} = 3.65 \frac{J}{m^2} \longrightarrow \Delta G^{*}_{dia} = 4.24 \times 10^{-18} J \rightsquigarrow I_{ratio} = 6.4 \times 10^{-5}$

for $\gamma_{dia} = 3.7 \frac{J}{m^2} \rightsquigarrow \Delta G^{*}_{dia} = 4.4 \times 10^{-18} \rightsquigarrow I_{ratio} = 4.3 \times 10^{13}$

g) it apears that although graphite is more stable in bulk state but by control of cluster size diamond can be obtained by CVD