Phase transformation 20192742 손수경

1. (a) $\frac{\partial C}{\partial t} = \tilde{D}\frac{\partial^2 C}{\partial x^2}$

$\frac{\partial C}{\partial t} = R(\beta)(12\text{at\%}, Au)\exp[R(\beta)t]\cos\beta x$

$\frac{\partial^2 C}{\partial x^2} = -\beta^2(12\text{at\%}, Au)\exp[R(\beta)t]\cos\beta x$

$\therefore R(\beta)(12\text{at\%}\,Au)\exp[R(\beta)t]\cos\beta x$

$= -\tilde{D}\beta^2(12\text{at\%}Au)\exp[R(\beta)t]\cos\beta(x)$ $\Rightarrow R(\beta) = -\tilde{D}\beta^2$

$C_{max} = (38\text{ at\%}\,Au) + (12\text{ at\%}\,Au)\exp[R(\beta)t]$ @ $\beta x = n\pi$ & n is even //

$C_{min} = (38\text{at\%}\,Au) - (12\text{at\%}\,Au)\exp(R(\beta)t)$ @ $\beta x = n\pi$ & n is odd

$C_{max} - C_{min} = 2(12\text{at\%}Au)\exp[R(\beta)t] = (2\text{at\%}\,Au)$

$12\exp[R(\beta)t] = 1 \Rightarrow 12\exp[-\tilde{D}\beta^2 t] = 1$

$$t = \frac{\ln(12)}{\tilde{D}\beta^2} = \frac{\ln(12)}{(10^{-23}m^2/s)\left(\frac{\pi}{10^{-9}m}\right)^2} \approx 25177s$$

(b) $\frac{\partial C}{\partial t} = \tilde{D}\frac{\partial^2 C}{\partial x^2} - \frac{2k\tilde{D}}{f''}\frac{\partial^4 C}{\partial x^4}$

$\frac{\partial^4 C}{\partial x^4} = \beta^4(12\text{at\%}\,Au)\exp[R(\beta)t]\cos\beta x$

$\Rightarrow R(\beta)(12\text{at\%}\,Au)\exp[R(\beta)t]\cos\beta x$

$= -\tilde{D}\beta^2(12\text{at\%}\,Au)\exp[R(\beta)t]\cos\beta x$

$-\frac{2k\tilde{D}}{f''}\beta^4(12\text{at\%}\,Au)\exp[R(\beta)t]\cos\beta x$

$R(\beta) = -\tilde{D}\beta^2 - \frac{2k\tilde{D}}{f''}\beta^4$

$$C_{max} - C_{min} = 2(12\,at\%\,Au)\exp[R(\beta)t] = 2\,at\%\,Au$$

$$12\exp[R(\beta)t] = 12\exp\left[\left(\tilde{D}\beta^2 - \frac{2K\tilde{D}}{f''}\beta^4\right)t\right] = 1$$

$$t = \frac{\ln(12)}{\tilde{D}\beta^2 + \frac{2K\tilde{D}}{f''}\beta^4} = \frac{\ln(12)}{(10^{-23}\,m^2/s)\left(\frac{\pi}{10^{-9}\,m}\right)^2 + \frac{2(-2.6\times10^{-11}\,J/m)(10^{-23}\,m^2s^{-1})}{(5\times10^9\,J/m^3)}\left(\frac{\pi}{10^{-9}\,m}\right)^4}$$

$\simeq 2805\,ns$

(c) Ag-Au system은 bond를 선호하고, negative gradient energy coefficient를 가짐.
계면 에너지를 고려한다면, spinodal decomposition을 덜 선호하는 (b)가 더 적절하다.