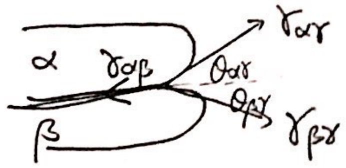
HW#6. 20182347 권현석

1.

(a) For the equilibrium of forces,



$$\Rightarrow \gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\theta_{\alpha\gamma} + \gamma_{\beta\gamma}\cos\theta_{\beta\gamma}$$

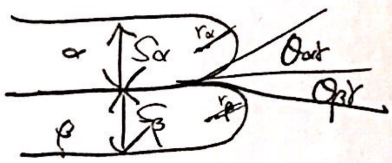
If the layers don't have the curvature,

$$\theta_{\alpha\gamma} = \theta_{\beta\gamma} = 90°$$

$$\Rightarrow \gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cdot 0 + \gamma_{\beta\gamma}\cdot 0 = 0,$$ which makes no sense

Therefore, the layers should have curvatures for a positive $\gamma_{\alpha\beta}$.

(b)



$$\begin{cases} S_\alpha = 2r_\alpha\cos\theta_{\alpha\gamma} & \cdots \text{i)} \\ S_\beta = 2r_\beta\cos\theta_{\beta\gamma} & \cdots \text{ii)} \end{cases}$$

For the equilibrium of forces,

$$\begin{cases} \gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\theta_{\alpha\gamma} + \gamma_{\beta\gamma}\cos\theta_{\beta\gamma} & \cdots \text{iii)} \\ \gamma_{\alpha\gamma}\sin\theta_{\alpha\gamma} = \gamma_{\beta\gamma}\sin\theta_{\beta\gamma}. & \cdots \text{iv)} \end{cases}$$

From iv), $\gamma_{\alpha\gamma}^2\sin^2\theta_{\alpha\gamma} = \gamma_{\beta\gamma}^2\sin^2\theta_{\beta\gamma}$

$$\gamma_{\alpha\gamma}^2(1-\cos^2\theta_{\alpha\gamma}) = \gamma_{\beta\gamma}^2(1-\cos^2\theta_{\beta\gamma})$$

$$\Rightarrow \cos\theta_{\beta\gamma} = \sqrt{\frac{\gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2\cos^2\theta_{\alpha\gamma} - \gamma_{\alpha\gamma}^2}{\gamma_{\beta\gamma}^2}}$$

From iii), iv),

$$\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\theta_{\alpha\gamma} + \sqrt{\gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2\cos^2\theta_{\alpha\gamma} - \gamma_{\alpha\gamma}^2}$$

From i),

$$\cos\theta_{\alpha\gamma} = \frac{S_\alpha}{2r_\alpha}$$

$$\Rightarrow \gamma_{\alpha\beta} = \frac{\gamma_{\alpha\gamma}S_\alpha}{2r_\alpha} + \sqrt{\gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\alpha\gamma}^2\frac{S_\alpha^2}{4r_\alpha^2}}$$

$$\gamma_{\alpha\beta}^2 - \frac{\gamma_{\alpha\gamma}\gamma_{\alpha\beta}S_\alpha}{r_\alpha} + \cancel{\frac{\gamma_{\alpha\gamma}^2S_\alpha^2}{4r_\alpha^2}} = \gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2 + \cancel{\gamma_{\alpha\gamma}^2\frac{S_\alpha^2}{4r_\alpha^2}}$$

$$\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2 = \frac{\gamma_{\alpha\gamma}\gamma_{\alpha\beta}S_\alpha}{r_\alpha}$$

$$\therefore r_\alpha = \frac{\gamma_{\alpha\gamma}\gamma_{\alpha\beta}S_\alpha}{\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2}$$

In the same way,

$$r_\beta = \frac{\gamma_{\beta\gamma}\gamma_{\alpha\beta}S_\beta}{\gamma_{\alpha\beta}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2}.$$

(c) ΔG due to the capillary effects:

$$\Delta G = \Delta G_\alpha + \Delta G_\beta$$

$$= \frac{\gamma_{\alpha\gamma}}{r_\alpha}V_\alpha + \frac{\gamma_{\beta\gamma}}{r_\beta}V_\beta$$

$$= \frac{\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2}{\gamma_{\alpha\beta}S_\alpha}\cdot V_\alpha + \frac{\gamma_{\alpha\beta}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2}{\gamma_{\alpha\beta}S_\beta}\cdot V_\beta$$

$$= \frac{\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2}{\gamma_{\alpha\beta}\cancel{S_\alpha}}\cdot\frac{\cancel{S_\alpha}V_m^l}{S} + \frac{\gamma_{\alpha\beta}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2}{\gamma_{\alpha\beta}\cancel{S_\beta}}\cdot\frac{\cancel{S_\beta}V_m^l}{S}$$

(from assuming that $\frac{V_m^l}{S} = \frac{V_\alpha}{S_\alpha} = \frac{V_\beta}{S_\beta}$)

$$= \frac{2\gamma_{\alpha\beta}}{S}V_m^l$$

$$= \Delta G_{IF}(S)$$