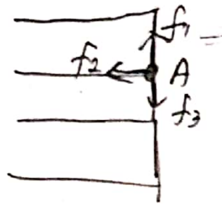
상변태론 Homework 6 20170370 이정준

1. (a) 만약 각 layer가 인접할때.

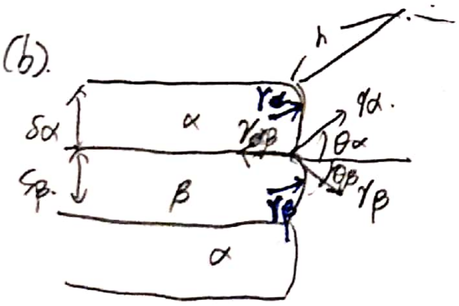
Curvature가 없다고 생각해보자.

그럼 각 layer는 아래처럼 flat한 surface를 가질것이다.



하지만 이경우는 매우 불안정하다. 3개의 plane이 만나는 junction A에서의 force balance를 고려해보면 surface 방향으로 f_1, f_2, f_3 총 3개의 tension이 존재한다. 이경우 force equilibrium을 만족하지 않는다. 즉 boundary는 net force 방향으로 움직이게 되고 이는 안쪽으로 말려들어가는 방향이므로 curvature를 갖게된다.

(b)



$r_\alpha \cos\theta_\alpha = \frac{S_\alpha}{2}$, $r_\beta \cos\theta_\beta = \frac{S_\beta}{2}$ (r_α, r_β : curvature of α, β layer)

junction에서의 force balance

$\Rightarrow$ $\gamma_\alpha \cos\theta_\alpha + \gamma_\beta \cos\theta_\beta = \gamma_{\alpha\beta}$
$\gamma_\alpha \sin\theta_\alpha = \gamma_\beta \sin\theta_\beta$ $\Rightarrow$ $\cos\theta_\alpha, \cos\theta_\beta$ 값을 알수있음

$\cos\theta_\alpha, \cos\theta_\beta$ 값을 위의 curvature 식에 대입하면,

$$r_\alpha = \frac{S_\alpha \gamma_\alpha \gamma_{\alpha\beta}}{\gamma_{\alpha\beta}^2 + \gamma_\alpha^2 - \gamma_\beta^2} \quad r_\beta = \frac{S_\beta \gamma_\beta \gamma_{\alpha\beta}}{\gamma_{\alpha\beta}^2 + \gamma_\beta^2 - \gamma_\alpha^2}$$
(r_α, r_β : curvature ; $\gamma_\alpha, \gamma_\beta, \gamma_{\alpha\beta}$: surface tension)

(c) length of tip growth : δ

$$\Delta G_{capillarity} = \frac{\gamma_\alpha}{r_\alpha} S_\alpha h\delta + \frac{\gamma_\beta}{r_\beta} S_\beta h\delta$$
$$= h\delta\left(\frac{\gamma_{\alpha\beta}^2 + \gamma_\alpha^2 - \gamma_\beta^2}{\gamma_{\alpha\beta}} + \frac{\gamma_{\alpha\beta}^2 + \gamma_\beta^2 - \gamma_\alpha^2}{\gamma_{\alpha\beta}}\right)$$
$$= 2\gamma_{\alpha\beta} \cdot h\delta.$$

$$\Delta G_{interface} = 2\gamma_{\alpha\beta} \cdot h\delta.$$

$\therefore$ $\Delta G_{capillarity} = \Delta G_{interface}$