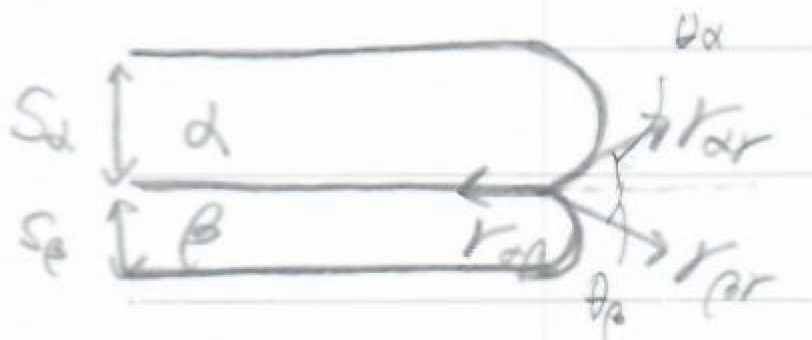
20192731 이동우

Date . . No.

1.

(a)



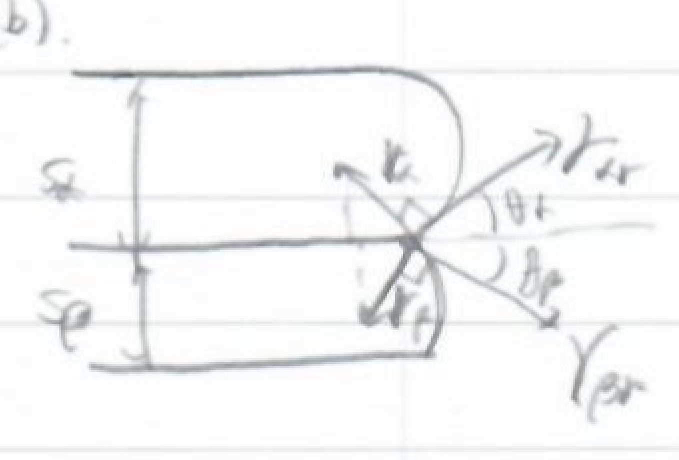
$\gamma_{\alpha\beta} = \gamma_{\alpha r}\cos\theta_\alpha + \gamma_{\beta r}\cos\theta_\beta$, $\gamma_{\alpha r}\cos\theta_\alpha = \gamma_{\beta r}\cos\theta_\beta$ ⇒ Force balance.

curvature가 없는 경우 $\theta_\alpha$와 $\theta_\beta$는 90°이고

위의 force balance 식에서 $\gamma_{\alpha\beta}$가 0이되기에 모순.

$\gamma_{\alpha\beta}$가 유한값을 갖기위해서는 $0 < \theta_\alpha, \theta_\beta < 90$.

(b).



$S_\alpha = 2r_\alpha\cos\theta_\alpha$ $S_\beta = 2r_\beta\cos\theta_\beta$

$\gamma_{\alpha r}\sin\theta_\alpha = \gamma_{\beta r}\sin\theta_\beta \Rightarrow \gamma_{\alpha r}^2\sin^2\theta_\alpha = \gamma_{\beta r}^2\sin^2\theta_\beta \Rightarrow \gamma_{\alpha r}^2(1-\cos^2\theta_\alpha) = \gamma_{\beta r}^2(1-\cos^2\theta_\beta)$

$\Rightarrow \cos\theta_\beta = \sqrt{1-\frac{\gamma_{\alpha r}^2}{\gamma_{\beta r}^2}(1-\cos^2\theta_\alpha)}$

$\gamma_{\alpha\beta} = \gamma_{\alpha r}\cos\theta_\alpha + \gamma_{\beta r}\cos\theta_\beta = \gamma_{\alpha r}\cos\theta_\alpha + \gamma_{\beta r}\sqrt{1-\frac{\gamma_{\alpha r}^2}{\gamma_{\beta r}^2}(1-\cos^2\theta_\alpha)}$

$\gamma_{\alpha\beta} - \gamma_{\alpha r}\cos\theta_\alpha = \gamma_{\beta r}\sqrt{1-\frac{\gamma_{\alpha r}^2}{\gamma_{\beta r}^2}(1-\cos^2\theta_\alpha)}$

$\gamma_{\alpha\beta}^2 - 2\gamma_{\alpha\beta}\gamma_{\alpha r}\cos\theta_\alpha + \gamma_{\alpha r}^2\cos^2\theta_\alpha = \gamma_{\beta r}^2 - \gamma_{\alpha r}^2(1-\cos^2\theta_\alpha) \Rightarrow \cos\theta_\alpha = \frac{\gamma_{\alpha\beta}^2 - \gamma_{\beta r}^2 + \gamma_{\alpha r}^2}{2\gamma_{\alpha\beta}\gamma_{\alpha r}}$

$\therefore r_\alpha = \frac{S_\alpha}{2\cos\theta_\alpha} = \frac{S_\alpha\gamma_{\alpha\beta}\gamma_{\alpha r}}{\gamma_{\alpha\beta}^2 - \gamma_{\beta r}^2 + \gamma_{\alpha r}^2}$ $r_\beta = \frac{S_\beta\gamma_{\alpha\beta}\gamma_{\beta r}}{\gamma_{\alpha\beta}^2 - \gamma_{\alpha r}^2 + \gamma_{\beta r}^2}$

(c)

$$\Delta G = \frac{r_{\alpha r}}{r_\alpha} V_\alpha + \frac{r_{\beta r}}{r_\beta} V_\beta = \left( \frac{S_\alpha r_{\alpha r}}{r_\alpha} + \frac{S_\beta r_{\beta r}}{r_\beta} \right) \frac{V_m^2}{S}, \quad \left( V_\alpha = \frac{S_\alpha}{S} V_m^2, \ V_\beta = \frac{S_\beta}{S} V_m^2 \right)$$

$$= \left\{ \frac{\cancel{S_\alpha r_{\alpha r}} (r_{\alpha\beta}^2 \cancel{- r_{\alpha r}^2 + r_{\alpha r}^2})}{\cancel{S_\alpha} r_{\alpha\beta} \cancel{r_{\alpha r}}} + \frac{\cancel{S_\beta r_{\beta r}} (r_{\alpha\beta}^2 \cancel{- r_{\beta r}^2 + r_{\beta r}^2})}{\cancel{S_\beta} r_{\alpha\beta} \cancel{r_{\beta r}}} \right\} \frac{V_m^2}{S}$$

$$= \frac{2 r_{\alpha\beta} V_m^2}{S} = \Delta G_{ZF}$$