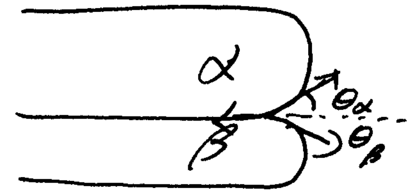
1.

a)



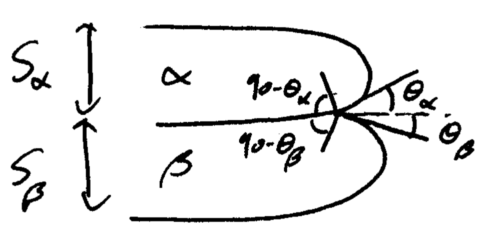
for equilibrium,

$\gamma_\alpha \cos\theta_\alpha + \gamma_\beta \cos\theta_\beta = \gamma_{\alpha\beta}$

if $\theta_\alpha = \theta_\beta = \frac{\pi}{2}$, $\gamma_{\alpha\beta}$ should be 0.

But any grain boundary can have 0 or negative γ.

b)



$r_\alpha \sin(90-\theta_\alpha) = \frac{S_\alpha}{2}$

$r_\beta \sin(90-\theta_\beta) = \frac{S_\beta}{2}$

$r_\alpha = \frac{S_\alpha}{2} \cdot \frac{1}{\cos\theta_\alpha}$

$r_\beta = \frac{S_\beta}{2} \frac{1}{\cos\theta_\beta}$

for equilibrium,

$\gamma_{\alpha\beta} = \gamma_\alpha \cos\theta_\alpha + \gamma_\beta \cos\theta_\beta$

$\gamma_\alpha \sin\theta_\alpha = \gamma_\beta \sin\theta_\beta$

$\gamma_{\alpha\beta} = \gamma_\alpha \sqrt{1-\sin^2\theta_\alpha} + \gamma_\beta \cos\theta_\beta$

$\gamma_{\alpha\beta} - \gamma_\beta \cos\theta_\beta = \gamma_\alpha \sqrt{1-\sin^2\theta_\alpha}$

$\gamma_{\alpha\beta}^2 - 2\alpha_{\alpha\beta}\gamma_\beta \cos\theta_\beta + \gamma_\beta^2 \cos^2\theta_\beta = \gamma_\alpha^2 (1-\sin^2\theta_\alpha)$

$= \gamma_\alpha^2 - \gamma_\beta^2 \sin^2\theta_\beta$

$\gamma_{\alpha\beta}^2 + \gamma_\beta^2 - \gamma_\alpha^2 = 2\gamma_{\alpha\beta}\gamma_\beta \cos\theta_\beta$

$$\cos\theta_\beta = \frac{\gamma_{\alpha\beta}^2 + \gamma_\beta^2 - \gamma_\alpha^2}{2\gamma_{\alpha\beta}\gamma_\beta}$$

$$\cos\theta_\alpha = \frac{\gamma_{\alpha\beta}^2 + \gamma_\alpha^2 - \gamma_\beta^2}{2\gamma_{\alpha\beta}\gamma_\alpha}$$

$$r_\alpha = \frac{S_\alpha}{2} \cdot \frac{2\gamma_{\alpha\beta}\cdot\gamma_\alpha}{\gamma_{\alpha\beta}^2 + \gamma_\alpha^2 - \gamma_\beta^2}$$

$$r_\beta = \frac{S_\beta}{2} \frac{2\gamma_{\alpha\beta}\gamma_\beta}{\gamma_{\alpha\beta}^2 + \gamma_\beta^2 - \gamma_\alpha^2}$$

C) $\cancel{\Delta P = \left(\frac{1}{r_1} + \frac{1}{r_2}\right)\gamma}$

~~if the growth like cly cylinder case~~

C) when the growth as cylinder,

$$\Delta G = \frac{\gamma_\alpha}{r_\alpha} V_\alpha + \frac{\gamma_\beta}{r_\beta} V_\beta$$

$$= \frac{(\gamma_{\alpha\beta}^2 + \gamma_\alpha^2 - \gamma_\beta^2)}{S_\alpha \gamma_{\alpha\beta}\gamma_\alpha} \cdot \gamma_\alpha V_\alpha + \frac{(\gamma_{\alpha\beta}^2 + \gamma_\beta^2 - \gamma_\alpha^2)}{S_\beta \gamma_{\alpha\beta}\gamma_\beta} \cdot \gamma_\beta V_\beta$$

assuming $\frac{V_\alpha}{S_\alpha} = \frac{V_\beta}{S_\beta} = \frac{V_m}{S}$ $\Rightarrow \Delta G = \frac{2\gamma_{\alpha\beta}V_m}{S}$