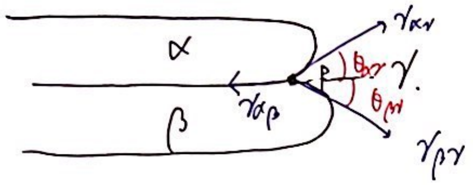
상변태론 HW 6 2020258f 권재성

1.

a) To earn the force balance at the interface.



We have total 3 interfaces. at point P.

$\gamma_{\alpha\beta}$. $\gamma_{\alpha\gamma}$. $\gamma_{\beta\gamma}$.

Drawing extension from the point P in the direction of $\gamma_{\alpha\beta}$. gives to angle

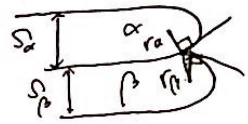
the force balance is written as.

i) $\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\theta_{\alpha\gamma} + \gamma_{\beta\gamma}\cos\theta_{\beta\gamma}$: $\gamma_{\alpha\beta}$ direction

ii) $\gamma_{\alpha\gamma}\sin\theta_{\alpha\gamma} = \gamma_{\beta\gamma}\sin\theta_{\beta\gamma}$. : normal to $\gamma_{\alpha\beta}$ direction

if the layer tip does not have curvature. the angle should be 90°.
then the equation is becomes $\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cdot 0 + \gamma_{\beta\gamma}\cdot 0 = 0$, which is impossible
So the layer tip should possess curvature.

b) Setting the curvature of α as r_α. and that of β as r_β.



$S_\alpha = 2r_\alpha\cos\theta_{\alpha\gamma}$, $S_\beta = 2r_\beta\cos\theta_{\beta\gamma}$. --- iii)

to express radius of curvature as a function of layer thickness we should replace θ with S and γ_s.

therefore $\gamma_{\alpha\gamma}\sin\theta_{\alpha\gamma} = \gamma_{\beta\gamma}\sin\theta_{\beta\gamma}$ $\therefore \gamma_{\alpha\gamma}^2\sin^2\theta_{\alpha\gamma} = \gamma_{\beta\gamma}^2\sin^2\theta_{\beta\gamma}$

$\therefore \gamma_{\alpha\gamma}^2(1-\cos^2\theta_{\alpha\gamma}) = \gamma_{\beta\gamma}^2(1-\cos^2\theta_{\beta\gamma}) \rightarrow \cos\theta_{\beta\gamma} = \sqrt{\dfrac{\gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\alpha\gamma}^2\cos^2\theta_{\alpha\gamma}}{\gamma_{\beta\gamma}^2}}$

using equation i) at problem (1.a)

$$\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\theta_{\alpha\gamma} + \gamma_{\beta\gamma}\sqrt{\frac{\gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\alpha\gamma}^2\cos^2\theta_{\alpha\gamma}}{\gamma_{\beta\gamma}^2}}$$

→

Substituting equation iii) $S_\alpha = 2r_\alpha \cos\theta_{\alpha\gamma}$, gives

$$\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\frac{S_\alpha}{2r_\alpha} + \sqrt{\gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\alpha\gamma}^2\frac{S_\alpha^2}{4r_\alpha^2}}$$

Square both sides gives. $\gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\alpha\gamma}^2\frac{S_\alpha^2}{4r_\alpha^2} = \gamma_{\alpha\beta}^2 - 2\gamma_{\alpha\beta}\gamma_{\alpha\gamma}\cdot\frac{S_\alpha}{2r_\alpha} + \gamma_{\alpha\gamma}^2\cdot\frac{S_\alpha^2}{4r_\alpha^2}$

$$\therefore \frac{1}{r_\alpha} = \frac{\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2}{S_\alpha\gamma_{\alpha\beta}\gamma_{\alpha\gamma}} \rightarrow \underline{r_\alpha = \frac{S_\alpha\gamma_{\alpha\beta}\gamma_{\alpha\gamma}}{\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2}}.$$

Without losing generality $\rightarrow$ $\underline{r_\beta = \frac{S_\beta\gamma_{\alpha\beta}\gamma_{\beta\gamma}}{\gamma_{\alpha\beta}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2}}.$

c) We've learned that Gibbs free energy via capillary effect in structure can be written as

$$\Delta G_\alpha = \frac{\gamma_{\alpha\gamma}\cdot V_{\alpha\gamma}}{r_\alpha}. \qquad \Delta G_\beta = \frac{\gamma_{\beta\gamma}V_{\beta\gamma}}{r_\beta}$$

$\therefore$ total Gibb's free energy is sum of ΔG_α & ΔG_β $\therefore \Delta G = \Delta G_\alpha + \Delta G_\beta$

$$\therefore \Delta G = \frac{\gamma_{\alpha\gamma}V_{\alpha\gamma}}{r_\alpha} + \frac{\gamma_{\beta\gamma}V_{\beta\gamma}}{r_\beta} = \frac{(\gamma_{\alpha\beta}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2)\gamma_{\alpha\gamma}V_\alpha}{S_\alpha\gamma_{\alpha\beta}\gamma_{\alpha\gamma}} + \frac{(\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2)\gamma_{\beta\gamma}V_\beta}{S_\beta\gamma_{\alpha\beta}\gamma_{\beta\gamma}}$$

assuming $\frac{V_\alpha}{S_\alpha} = \frac{V_\beta}{S_\beta} = \frac{V_m}{S}$

$$\boxed{\Delta G = \frac{2\gamma_{\alpha\beta}V_m}{S}}$$