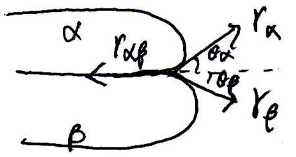
Problem Set # 6 20212000 철강·에너지소재 조영근

a)



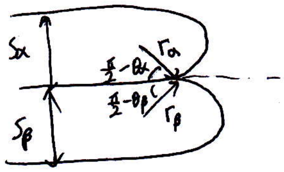
Force balance에 따라 $\gamma_{\alpha\beta} = \gamma_\alpha \cos\theta_\alpha + \gamma_\beta \cos\theta_\beta$ …①, $\gamma_\alpha \sin\theta_\alpha = \gamma_\beta \sin\theta_\beta$ …②

만약 curvature가 없다고 가정한다면 ($\theta_\alpha = \theta_\beta = 90^\circ$)

①에서 $\gamma_{\alpha\beta} = 0$ 이 되어 모순된 결과이다.

따라서 각 layer는 curvature를 가진다.

b)



$$r_\alpha \sin\left(\frac{\pi}{2}-\theta_\alpha\right) = \frac{1}{2}S_\alpha = r_\alpha \cos\theta_\alpha \rightarrow r_\alpha = \frac{S_\alpha}{2\cos\theta_\alpha} \quad \cdots ③$$

$$r_\beta \sin\left(\frac{\pi}{2}-\theta_\beta\right) = \frac{1}{2}S_\beta = r_\beta \cos\theta_\beta \rightarrow r_\beta = \frac{S_\beta}{2\cos\theta_\beta} \quad \cdots ④$$

따라서 ①,②에서 $\cos\theta_\alpha$, $\cos\theta_\beta$를 구하여 ③,④에 대입하면 된다.

$$② \rightarrow \gamma_\alpha^2 \sin^2\theta_\alpha = \gamma_\beta^2 \sin^2\theta_\beta$$

$$\gamma_\alpha^2(1-\cos^2\theta_\alpha) = \gamma_\beta^2(1-\cos^2\theta_\beta)$$

$$\cos\theta_\beta = \sqrt{1-\frac{\gamma_\alpha^2}{\gamma_\beta^2}(1-\cos^2\theta_\alpha)} \quad \cdots ⑤$$

⑤를 ①에 대입 → $\gamma_{\alpha\beta} = \gamma_\alpha \cos\theta_\alpha + \gamma_\beta \sqrt{1-\frac{\gamma_\alpha^2}{\gamma_\beta^2}(1-\cos^2\theta_\alpha)}$

$$(\gamma_{\alpha\beta} - \gamma_\alpha\cos\theta_\alpha)^2 = \left(\gamma_\beta\sqrt{1-\frac{\gamma_\alpha^2}{\gamma_\beta^2}(1-\cos^2\theta_\alpha)}\right)^2$$

$$\gamma_{\alpha\beta}^2 + \cancel{\gamma_\alpha^2\cos^2\theta_\alpha} - 2\gamma_{\alpha\beta}\gamma_\alpha\cos\theta_\alpha = \gamma_\beta^2 - \gamma_\alpha^2 + \cancel{\gamma_\alpha^2\cos^2\theta_\alpha}$$

$$\cos\theta_\alpha = \frac{\gamma_{\alpha\beta}^2 - \gamma_\beta^2 + \gamma_\alpha^2}{2\gamma_{\alpha\beta}\gamma_\alpha} \quad \cdots ⑥$$

⑥을 ③에 대입 → $r_\alpha = \dfrac{S_\alpha \gamma_{\alpha\beta}\gamma_\alpha}{\gamma_{\alpha\beta}^2 - \gamma_\beta^2 + \gamma_\alpha^2}$ …⑦

같은 방법으로 θ_β에 대해 진행하면 $r_\beta = \dfrac{S_\beta \gamma_{\alpha\beta}\gamma_\beta}{\gamma_{\alpha\beta}^2 - \gamma_\alpha^2 + \gamma_\beta^2}$ …⑧

c) Capillary effect로 인한 $\Delta G = \frac{\gamma_\alpha}{r_\alpha} V_\alpha + \frac{\gamma_\beta}{r_\beta} V_\beta$ ⋯ ⑨

$V_\alpha = \frac{S_\alpha}{S} V_m^L$, $V_\beta = \frac{S_\beta}{S} V_m^L$ 으로 가정하여 ⑨에 대입,

⑦, ⑧도 ⑨에 대입

$$\rightarrow \Delta G = \frac{\gamma_{\alpha\beta}^2 - \gamma_\beta^2 + \gamma_\alpha^2}{\cancel{S_\alpha} \gamma_{\alpha\beta}} \cdot \frac{\cancel{S_\alpha}}{S} V_m^L + \frac{\gamma_{\alpha\beta}^2 - \gamma_\alpha^2 + \gamma_\beta^2}{\cancel{S_\beta} \gamma_{\alpha\beta}} \cdot \frac{\cancel{S_\beta}}{S} V_m^L$$

$$= \frac{V_m^L}{\gamma_{\alpha\beta} S} \cdot 2\gamma_{\alpha\beta}^2$$

$$= \underline{\frac{2\gamma_{\alpha\beta}}{S} V_m^L}$$ (α/β interfacial energy를 고려하여 얻은 값과 동일)