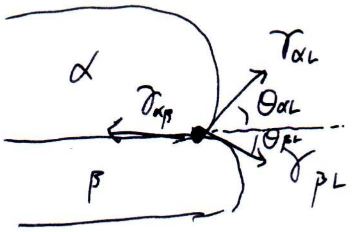
1. a) By surface tension:



$$\begin{cases} \gamma_{\alpha\beta} = \gamma_{\alpha L}\cos\theta_{\alpha L} + \gamma_{\beta L}\cos\theta_{\beta L} \\ \gamma_{\alpha L}\sin\theta_{\alpha L} = \gamma_{\beta L}\sin\theta_{\beta L} \end{cases}$$

$$\{\theta_{\alpha L} = 0 \leftrightarrow \theta_{\beta L} = 0\} \leftrightarrow \gamma_{\alpha\beta} = \gamma_{\alpha L} + \gamma_{\beta L}$$

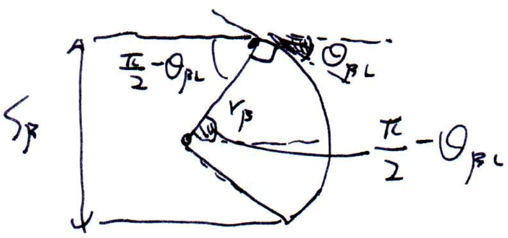
$$\theta_{\alpha L} = \frac{\pi}{2} \rightarrow \begin{cases} \gamma_{\alpha\beta} = \gamma_{\beta L}\cos\theta_{\beta L} \\ \gamma_{\alpha L} = \gamma_{\beta L}\sin\theta_{\beta L} \end{cases} \rightarrow \gamma_{\alpha\beta} < \gamma_{\beta L}$$

$$\theta_{\beta L} = \frac{\pi}{2} \rightarrow \begin{cases} \gamma_{\alpha\beta} = \gamma_{\alpha L}\cos\theta_{\alpha L} \\ \gamma_{\beta L} = \gamma_{\alpha L}\sin\theta_{\alpha L} \end{cases} \rightarrow \gamma_{\alpha\beta} < \gamma_{\alpha L}$$

(P.) in real, $\gamma_{\alpha\beta}$ often much bigger than $\gamma_{\alpha-L}$, $\gamma_{\beta-L}$.

$\therefore \theta_{\alpha L} \neq \frac{\pi}{2}$, $\theta_{\beta L} \neq \frac{\pi}{2}$, generally.

b)



$$\frac{1}{2}S_\beta = r_\beta \cdot \sin\left(\frac{\pi}{2} - \theta_{\beta L}\right)$$
$$= r_\beta \cos\theta_{\beta L}$$

$$\begin{cases} \gamma_{\alpha\beta} = \gamma_{\alpha L}\cos\theta_{\alpha L} + \gamma_{\beta L}\cos\theta_{\beta L} \\ \gamma_{\alpha L}\sin\theta_{\alpha L} = \gamma_{\beta L}\sin\theta_{\beta L} \end{cases}$$

$$\rightarrow \gamma_{\alpha L}^2 \underbrace{\sin^2\theta_{\alpha L}}_{=1-\cos^2\theta_{\alpha L}} = \gamma_{\beta L}^2 \sin^2\theta_{\beta L}.$$

$$\cos\theta_{\alpha L} = \left[1 - \left(\frac{\gamma_{\beta L}}{\gamma_{\alpha L}}\right)^2 (1-\cos^2\theta_{\beta L})\right]^{\frac{1}{2}}$$

$$(\gamma_{\alpha\beta} - \gamma_{\beta L}\cos\theta_{\beta L})^2 = (\gamma_{\alpha L}\cos\theta_{\alpha L})^2$$

$$\begin{cases} \text{left hand side} = \gamma_{\alpha\beta}^2 + \gamma_{\beta L}^2\cos^2\theta_{\beta L} - 2\gamma_{\alpha\beta}\gamma_{\beta L}\cos\theta_{\beta L} \\ \text{right hand side} = \gamma_{\alpha L}^2\left[1 - \left(\frac{\gamma_{\beta L}^2}{\gamma_{\alpha L}^2}\right)(1-\cos^2\theta_{\beta L})\right] \end{cases}$$
$$= \gamma_{\alpha L}^2 - \gamma_{\beta L}^2(1-\cos^2\theta_{\beta L})$$
$$= \gamma_{\alpha L}^2 + \gamma_{\beta L}^2\cos^2\theta_{\beta L} - \gamma_{\beta L}^2$$

$$\rightarrow -2\gamma_{\alpha\beta}\gamma_{\beta L}\cos\theta_{\beta L} = \gamma_{\alpha L}^2 - \gamma_{\beta L}^2 - \gamma_{\alpha\beta}^2$$

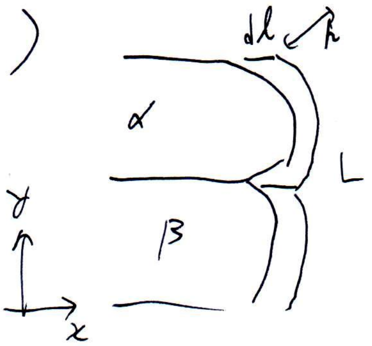
$$\therefore \cos\theta_{\beta L} = \frac{\gamma_{\alpha\beta}^2 - \gamma_{\alpha L}^2 + \gamma_{\beta L}^2}{2\gamma_{\alpha\beta}\gamma_{\beta L}}$$

$$\frac{1}{2} S_\beta = r_\beta \cos\theta_{\beta L}$$

$$\therefore\ r_\beta = \frac{S_\beta}{2\cos\theta_{\beta L}}$$

$$= S_\beta \cdot \frac{\gamma_{\alpha\beta}\gamma_{\beta L}}{\gamma_{\alpha\beta}^2 - \gamma_{\alpha L}^2 + \gamma_{\beta L}^2}. \qquad \text{w.l.o.g.}\ r_\alpha = S_\alpha \cdot \frac{\gamma_{\alpha\beta}\gamma_{\alpha L}}{\gamma_{\alpha\beta}^2 + \gamma_{\alpha L}^2 - \gamma_{\beta L}^2}$$

c)



Let assume no growth in y direction..

$$dF = -\sum S_i\, dT - \sum P_i\, dV_i + \sum \gamma_i\, dA_i$$

$$\rightsquigarrow\ \Delta P^{\alpha}_{cap} = \frac{\gamma_{\alpha\beta}\cdot(2dl\cdot h)/2}{dV_{\alpha-L}} \leftarrow 2\ \alpha/\beta \text{ interfaces}$$

$$dV_{\alpha-L} = S_\alpha \cdot dl \cdot h$$

$$= \frac{\gamma_{\alpha\beta}\cancel{dl\cdot h}}{S_\alpha \cancel{dl\cdot h}}$$

$$\Delta G_{cap} = \sum V_i \Delta P_i$$

$$= V_{\alpha,m}\cdot\frac{\gamma_{\alpha\beta}}{S_\alpha} + V_{\beta,m}\frac{\gamma_{\alpha\beta}}{S_\beta} \leftarrow \text{let } V_{\alpha,m} = \left(\frac{S_\alpha}{S}\right)\cdot V_m^L,\quad V_{\beta,m} = \left(\frac{S_\beta}{S}\right) V_m^L$$

$$= \gamma_{\alpha\beta} V_m^L \cdot \left(\frac{1}{\cancel{S_\alpha}}\cdot\frac{\cancel{S_\alpha}}{S} + \frac{1}{S_\beta}\cdot\frac{S_\beta}{S}\right)$$

$$= \frac{2\gamma_{\alpha\beta} V_m^L}{S}$$

$$\therefore\ \Delta G_{cap} = \Delta G_{IF}$$