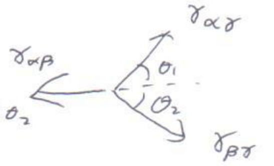
phase Transformation Problem set #6

20192259 신소재 왕재민

1-a) Surface tension 들의 force balance 에 의해서 각 layer에 curvature가 생긴다.

α 와 β, γ 가 만나는 점을 대략적으로 그리면, 오른쪽과 같다.

force balance equation 은

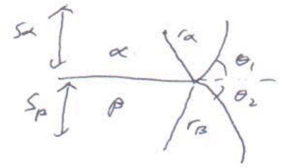


$$\begin{cases} \gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\theta_1 + \gamma_{\beta\gamma}\cos\theta_2 \\ \gamma_{\alpha\gamma}\sin\theta_1 = \gamma_{\beta\gamma}\sin\theta_2 \end{cases}$$

만약 curvature 가 없으면, 즉 $\theta_1 = \theta_2 = 90°$ 라면, $\gamma_{\alpha\beta} = 0$ 이 되므로, 모순이다. 따라서, 각 layer의 끝은 curvature 가 생기는 것이 자연스럽다.

1-b) $2r_\alpha \sin\left(\frac{\pi}{2} - \theta_1\right) = S_\alpha = 2r_\alpha\cos\theta_1$

$2r_\beta \sin\left(\frac{\pi}{2} - \theta_2\right) = S_\beta = 2r_\beta\cos\theta_2$



$$r_\alpha = \frac{S_\alpha}{2\cos\theta_1} \qquad r_\beta = \frac{S_\beta}{2\cos\theta_2}$$

Force balance 에서,

$$\begin{cases} \gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\theta_1 + \gamma_{\beta\gamma}\cos\theta_2 \\ \gamma_{\alpha\gamma}\sin\theta_1 = \gamma_{\beta\gamma}\sin\theta_2 \end{cases}$$

$$\sin\theta_1 = \frac{\gamma_{\beta\gamma}}{\gamma_{\alpha\gamma}}\sin\theta_2 \qquad \gamma_{\alpha\beta} = \gamma_{\alpha\gamma}(1-\sin^2\theta_1)^{\frac{1}{2}} + \gamma_{\beta\gamma}\cos\theta_2$$

$$\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\left(1 - \frac{\gamma_{\beta\gamma}^2}{\gamma_{\alpha\gamma}^2}\sin^2\theta_2\right)^{\frac{1}{2}} + \gamma_{\beta\gamma}\cos\theta_2$$

$$\gamma_{\alpha\beta}^2 - 2\gamma_{\alpha\beta}\gamma_{\beta\gamma}\cos\theta_2 + \gamma_{\beta\gamma}^2\cos^2\theta_2 = \gamma_{\alpha\gamma}^2\left(1 - \frac{\gamma_{\beta\gamma}^2}{\gamma_{\alpha\gamma}^2}\sin^2\theta_2\right)$$

$$\gamma_{\alpha\beta}^2 - 2\gamma_{\alpha\beta}\gamma_{\beta\gamma}\cos\theta_2 + \gamma_{\beta\gamma}^2\cos^2\theta_2 = \gamma_{\alpha\gamma}^2 - \gamma_{\beta\gamma}^2\sin^2\theta_2$$

$$\cos\theta_2 = \frac{\gamma_{\alpha\beta}^2 + \gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2}{2\gamma_{\alpha\beta}\gamma_{\beta\gamma}}$$

같은 방법으로, $\cos\theta_1 = \dfrac{\gamma_{\alpha\beta}^2 + \gamma_{\alpha\gamma}^2 - \gamma_{\beta\gamma}^2}{2\gamma_{\alpha\beta}\gamma_{\alpha\gamma}}$

따라서, $r_\alpha = \dfrac{S_\alpha\gamma_{\alpha\beta}\gamma_{\alpha\gamma}}{\gamma_{\alpha\beta}^2 + \gamma_{\alpha\gamma}^2 - \gamma_{\beta\gamma}^2}$ $\quad r_\beta = \dfrac{S_\beta\gamma_{\alpha\beta}\gamma_{\beta\gamma}}{\gamma_{\alpha\beta}^2 + \gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2}$

1-c) $$\Delta G_{capillary} = \frac{\gamma_{\alpha\gamma}V_\alpha}{r_\alpha} + \frac{\gamma_{\beta\gamma}V_\beta}{r_\beta} = \frac{V_\alpha(\gamma_{\alpha\beta}^2 + \gamma_{\alpha\gamma}^2 - \gamma_{\beta\gamma}^2)}{S_\alpha\gamma_{\alpha\beta}} + \frac{V_\beta(\gamma_{\alpha\beta}^2 + \gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2)}{S_\beta\gamma_{\alpha\beta}}$$

$\dfrac{V_\alpha}{S_\alpha} = \dfrac{V_\beta}{S_\beta} = \dfrac{V_m}{S}$ 으로 가정하면,

$$\Delta G_{capillary} = \frac{2\gamma_{\alpha\beta}V_m}{S} = \Delta G_{IF}$$