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1.
$$\Delta G_{total} = \Delta G_1 \cdot \left(-\frac{4}{3}\pi r^3\right) + 4\pi r^2 \cdot \gamma$$

For nuclei that contains n atom, its volume is $n \cdot v$ where v is atomic volume.

so, $\frac{4}{3}\pi r^3 = nv$

$r = \left(\frac{3nv}{4\pi}\right)^{1/3}$, Surface Area $A = 4\pi r^2 = 4\pi \cdot \frac{3^{2/3} n^{2/3} v^{2/3}}{4^{2/3}\pi^{2/3}}$

$= (36\pi)^{1/3} \cdot n^{2/3} \cdot v^{2/3}$.

$$\Delta G_{total} = -\Delta G \cdot nV + (36\pi)^{1/3} \cdot n^{2/3} \cdot V^{2/3}$$

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2. a) $\Delta G = -nV \cdot \Delta G_v + (36\pi)^{1/3} n^{2/3} \cdot V^{2/3} \cdot \gamma$

b) $\frac{d\Delta G}{dn} = -V \cdot \Delta G_v + (36\pi)^{1/3} \cdot \frac{2}{3} \cdot n^{-1/3} \cdot V^{2/3} \cdot \gamma = 0$

$(V\Delta G_v)^3 = \frac{\cancel{36}\,4\pi}{n} \cdot V^2 \cdot \frac{8}{\cancel{27}\,3}\gamma^3 = \frac{32\pi V^2 \gamma^3}{3n}$

$\boxed{n = \frac{32\pi\gamma^3}{3V\Delta G_v^3}}$, $\Delta G = -\frac{32\pi\gamma^3}{3V\Delta G_v^3} \cdot V \cdot \Delta G_v + (36\pi)^{1/3} \cdot \left(\frac{32\pi\gamma^3}{3V\Delta G_v^3}\right)^{2/3} V^{2/3} \gamma$

$= -\frac{32\pi\gamma^3}{\Delta G_v^2} + 36\pi \cdot \frac{\gamma^3}{\Delta G_v^2} = .$

2\. a) $\Delta G_t = -nV\Delta G_v + (36\pi)^{1/3} \cdot n^{2/3} V^{2/3} \cdot \gamma$

b)
$$\frac{d\Delta G_t}{dn} = 0 = -V\Delta G_v + (36\pi)^{1/3} \cdot n^{-1/3} \cdot v^{2/3} \cdot \gamma \cdot \left(\frac{2}{3}\right)^3$$

$$(V\Delta G_v)^3 = \overset{\cancel{12}4}{\cancel{36}}\pi \cdot \frac{1}{n} \cdot v^2 \cdot \gamma^3 \cdot \cancel{3} \cdot \frac{8}{\cancel{27}\,3}$$

$$n = \frac{32\pi V^2 \gamma^3}{3V^3 \Delta G_v^3} = \frac{32\pi\gamma^3}{3V\Delta G_v^3}$$

$$\Delta G_t = -\frac{32\pi\gamma^3}{3V\Delta G_v^3} V \cdot \Delta G_v + (36\pi)^{1/3} \cdot \cancel{n^{2/3}} \cdot v^{2/3} \cdot \gamma \cdot \left(\frac{32\pi\gamma^3}{3V\Delta G_v^3}\right)^{2/3}$$

$$= -\frac{32\pi\gamma^3}{3\Delta G_v^2} + \left(\frac{\overset{4}{\cancel{36}}\pi \cdot \cancel{v^2} \cdot \gamma^3 \cdot (32)^2 \cdot \pi^2 \gamma^6}{\cancel{9}\cancel{V^2}\Delta G_v^6}\right)^{1/3}$$

$$= -\frac{32\pi\gamma^3}{3\Delta G_v^2} + \left(\frac{2^{12} \cdot \gamma^3 \cdot \pi^3 \cdot \gamma^6}{\Delta G_v^6}\right)^{1/3}$$

$$= -\frac{32\pi\gamma^3}{3\Delta G_v^2} + \frac{16\pi\gamma^3}{\Delta G_v^2} = \frac{16\pi\gamma^3}{3\Delta G_v^2}$$

c)

~~dr~~ assuming $\gamma_{dia} = 3.6\ J/m^2$

$$\Delta G_{gra} = -n({}^\circ G_v - {}^\circ G_{gra}) + (36\pi)^{1/3} n^{2/3} V_{gra}^{2/3} \gamma_{gra}$$

$$-\quad \Delta G_{dia} = -n({}^\circ G_v - {}^\circ G_{dia}) + (36\pi)^{1/3} n^{2/3} V_{dia}^{2/3} \gamma_{dia}$$

$$\Delta G_{gra} - \Delta G_{dia} = -n({}^\circ G_{dia} - {}^\circ G_{gra}) + (36\pi)^{1/3} n^{2/3} \cdot \cancel{(V_{gra}^{2/3} - V_{dia}^{2/3}) \cdot (\gamma_{gra} - \gamma_{dia})} = 0$$

$$(V_{gra}^{2/3}\gamma_{gra} - V_{dia}^{2/3} \cdot \gamma_{dia})$$

$$n = \frac{36\pi\left(V_{gra}^{2/3}\cdot\gamma_{gra} - V_{dia}^{2/3}\gamma_{dia}\right)^3}{\left({}^{\circ}G_{dia} - {}^{\circ}G_{gra}\right)^3}$$

$$= \frac{36\pi\cdot\left((8\times10^{-30})^{2/3}\cdot 3.1 - (6\times10^{-30})^{2/3}\cdot 3.6\right)^3}{\left(0.02\times1.6\times10^{-19}\right)^3} \doteqdot 466$$

if) $\gamma_{dia} = 3.65\,J/m^2 \rightarrow n \doteqdot 145$

$\gamma_{dia} = 3.7\,J/m^2 \rightarrow n \doteqdot \cancel{21.7}\ 21$

d) To stabilize the diamond for any ~~and~~ size condition,

$\Delta G_{dia} - \Delta G_{gra} < 0$ for any size

$$\Delta G_{gra} - \Delta G_{dia} = -n\left({}^{\circ}G_{dia} - {}^{\circ}G_{gra}\right) + (36\pi)^{1/3}\cdot n^{2/3}\cdot\left(V_{gra}^{2/3}\gamma_{gra} - V_{dia}^{2/3}\gamma_{dia}\right)$$

to $\Delta G_{dia} - \Delta G_{gra} < 0$ for any size, every term in the right side should be ~~neg~~ have positive ~~negative~~ value.

i) ${}^{\circ}G_{dia} - {}^{\circ}G_{gra} < 0$, ii) $V_{gra}^{2/3}\gamma_{gra} - V_{dia}^{2/3}\gamma_{dia} > 0$

e) $n^* = \frac{32\pi}{3V}\left(\frac{\gamma}{\Delta G_V}\right)^3 = 100$

$\underline{\Delta G_V = 1.08\times10^{10}\,J/m^3}$

f) $\dfrac{I_{gra}}{I_{dia}} = \dfrac{A\exp(-\Delta G^*_{gra}/kT)}{A\exp(-\Delta G^*_{dia}/kT)} = \exp\left(-\dfrac{\Delta G_{gra}-\Delta G_{dia}}{kT}\right)$

$\Delta G_{gr}^* = \dfrac{16\pi}{3}\dfrac{\gamma_{gr}^3}{(\Delta G_{vgr})^2} = 4.3\times10^{-18}\,J$

$\Delta G_{dia} = 4.1\times10^{-18}\,J$ for $\gamma = 3.6\,J/m^2$ case

$4.28\times10^{-18}\,J$ for $\gamma = 3.65\,J/m^2$ case

$4.46\times10^{-18}\,J$ for $\gamma = 3.7\,J/m^2$ case

assume that $T = 300k$,

$\dfrac{I_{gra}}{I_{dia}} = 3.42\times10^{-21}$ for $\gamma = 3.6\,J/m^2$

5.20×10^{-3} for $\gamma = 3.65\,J/m^2$

2.61×10^{16} for $\gamma = 3.7\,J/m^2$

(g) normaly, graphite is known as a more stable material than diamond. but in nanoscale, below the critical size, diamond can be more stable than graphite.

Furthermore, if there is some way to reduce surface energy of diamond, critical size would be greatly decreased.

h) CH_4 would be decomposed into C and H_2.
if C concentration during nucleation is above critical ~~radius~~ value, graphite can be stable form.
but if C concentration is below critical value, it would be formed as a diamond.
So, if graphite was formed, its driving force is reduction of capilary force due to large size of particle, we can reduce CH_4 concentration or T to lower supply of CH_4.