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1 (a) homogenous nucleation, nucleus → spherical.

total Gibbs free energy change in system ⇒ ΔG

$\Delta G = +\frac{4}{3}\pi r^3 \Delta G_v + 4\pi r^2 \gamma \quad (\Delta G_v < 0)$

at critical radius r^* → $\frac{d\Delta G}{dr}\Big|_{r=r^*} = 0$

$\frac{d\Delta G}{dr} = +4\pi r^2 \Delta G_v + 8\pi r \gamma = 0 \quad r = r^* = -\frac{2\gamma}{\Delta G_v}$

energy barrier for nucleation (ΔG^*) = $\frac{16\pi\gamma^3}{3(\Delta G_v)^2}$

Homogenous nucleation rate(I) is propotionally increase with the number of nuclei size with r^*.

$\Rightarrow I = f_0 C_0 \exp\left(-\frac{\Delta G^*}{kT}\right)$ boltzman distribution.

$\ln I \propto -\frac{\Delta G^*}{kT}$ 기울기 $= -\frac{\Delta G^*}{k} = -\frac{16\pi\gamma^3}{3k(\Delta G_v)^2} = -23.8\times10^3$

$\gamma = 0.27 J/m^2$

(b) $r^* = \frac{2\gamma}{-\Delta G_v} = -\frac{2\times 0.27 J/m^2}{-10^8 J/m^3} = 5.4\times10^{-9} m.$

(c) $V_{nuclei} = \frac{4}{3}\pi r^{*3} \quad V_{tin.atom} = \frac{4}{3}\pi r_{atom}^3.$

$\frac{V_{nuclei}}{V_{atom}} = \frac{(5.4\times10^{-9})^3}{(1.5\times10^{-10})^3} = 46656$ atoms/nuclei

2. Cubic particle이 생겨날때, System의 total Gibbs free E 변화 = ΔG

$\Delta G = -V\Delta G_v + \Sigma A_i \gamma_i$

$\Rightarrow \Delta G = -l^2 h \Delta G_v + l^2(\gamma_{sc} + \gamma_{cv} - \gamma_{sv}) + 4lh\gamma_{cv}$

Assume constant volume. $V = l^2 h \rightarrow \frac{dh}{dl} = -\frac{2h}{l}$

$\frac{\partial \Delta G}{\partial l} = -2lh\Delta G_v - l^2\frac{dh}{dl}\Delta G_v + 2l(\gamma_{sc} + \gamma_{cv} - \gamma_{sv}) + 4h\gamma_{cv} + 4l\frac{dh}{dl}\gamma_{cv} = 0 \quad (l = l^*)$

$0 = \cancel{Al\Delta G_v} + 2l(\gamma_{sc} + \gamma_{cv} - \gamma_{sv}) - 4h\gamma_{cv}$

$\left(\frac{l^*}{h}\right) = \frac{2\gamma_{cv}}{\gamma_{sc} + \gamma_{cv} - \gamma_{sv}}$

$l = \frac{2\gamma_{cv}}{\gamma_{sc} + \gamma_{cv} - \gamma_{sv}} = \frac{2\gamma_{cv}}{A}h$

$\Delta G = -\Delta G_v \cdot \frac{A}{2\gamma_{cv}} l^3 + Al^2 + 2Al^2 = -\Delta G_v \cdot \frac{A}{2\gamma_{cv}} l^3 + 3Al^2$

$\frac{d\Delta G}{dl} = \frac{-3A\Delta G_v}{2\gamma_{cv}} l^2 + 6Al = 0 \quad l^* = \frac{4\gamma_{cv}}{\Delta G_v}$

$h^* = \frac{Al}{2\gamma_{cv}} = \frac{2(\gamma_{sc} + \gamma_{cv} - \gamma_{sv})}{\Delta G_v}$

$\Delta G^* = \frac{16\gamma_{cv}^2}{\Delta G_v^2}(\gamma_{sc} + \gamma_{cv} - \gamma_{sv})$