상변태 HW4 2020210ㅁㅁ 권재선

1. $\Delta G = -\Delta G_v \cdot V + \sum A_i \gamma_i = -\Delta G_v \cdot V + 4\pi r^2 \cdot \gamma$ $V = \frac{4}{3}\pi r^3$

$\therefore \frac{d\Delta G}{dr} = -\Delta G_v \cdot 4\pi r^2 + 8\pi r \gamma = 0$ $r^* = \frac{2\gamma}{\Delta G_v}$ $\Delta G^* = \frac{16\pi\gamma^3}{3\Delta G_v}$.

Using Maxwell-Boltzmann distribution

$n = n_0 \exp\left(-\frac{Q}{kT}\right)$. $\ln n - \ln n_0 = -\frac{\Delta G}{k} \cdot \frac{1}{T}$

Substituting critical free energy value gives

a) Slope: $-\frac{\Delta G}{k} = -23.8 \times 10^3 K$ $\therefore \gamma^3 = \frac{\text{Slope} \times k \cdot 3 \cdot \Delta G_v^2}{16\pi} = 1.9602 \times 10^{-4}$

$\therefore \gamma = 5.8 \times 10^{-2} J/m^2$

b) $r^* = \frac{2\gamma}{\Delta G_v} = 1.16 \times 10^{-9} m = 1.16 nm$.

c) $\Delta G = -nV \cdot \Delta G_v + (36\pi)^{1/3} \cdot n^{2/3} \cdot v^{2/3} \cdot \gamma$

from critical ΔG value $\left.\frac{d\Delta G}{dn}\right|_{n=n^*} = 0$.

$\therefore \frac{d\Delta G}{dn^*} = -V \Delta G_v + \frac{2}{3}(36\pi)^{1/3} v^{2/3} \cdot n^{-\frac{1}{3}} \gamma$ $\therefore n^* = 465$

2. The Gibbs free energy change can be written as below.

$$\Delta G = -\Delta G_v \cdot V + \sum_i A_i \gamma_i \qquad V = l^2 h,$$

New ~~sur~~ interface: $l^2 \cdot (\gamma_{sc} + \gamma_{cv}) + 4lh \cdot \gamma_{cv}$

disappeared interface: $-l^2 \cdot \gamma_{vs}$

∴ the equation is

$$\Delta G = -\Delta G_v \cdot l^2 h + l^2 (\gamma_{sc} + \gamma_{cv} - \gamma_{vs}) + 4lh\gamma_{cv}.$$

Near the critical size assume V: const.

$$0 = 2lh + l^2 \cdot \frac{\partial h}{\partial l} \qquad \therefore \underline{\frac{\partial h}{\partial l} = -\frac{2h}{l}}$$

the partial differentiate of Gibb's free energy with l. gives the equation

$$\frac{\partial \Delta G}{\partial l} = -2\Delta G_v l \cdot h - \Delta G_v \cdot l^2 \cdot \frac{\partial h}{\partial l} + 2l(\gamma_{sc} + \gamma_{cv} - \gamma_{vs}) + 4h\gamma_{cv} + 4l\gamma_{cv} \cdot \frac{\partial h}{\partial l} = 0.$$

at critical size

$$0 = -2\Delta G_v lh + 2\Delta G_v \cdot lh + 2l(\gamma_{sc} + \gamma_{cv} - \gamma_{vs}) + 4h\gamma_{cv} - 8h\gamma_{cv}$$

$$\therefore 4h\gamma_{cv} = 2(\gamma_{sc} + \gamma_{cv} - \gamma_{vs})\, l \qquad \therefore \boxed{l = \frac{2\gamma_{cv}}{\gamma_{cv} + \gamma_{sc} - \gamma_{vs}} h}.$$

setting $\gamma_{cv} + \gamma_{sc} - \gamma_{vs} = \alpha$. $\quad l = \frac{2\gamma_{cv}}{\alpha} \cdot h$

substituting $l = \frac{2\gamma_{cv}}{\alpha} \cdot h$ to Gibb's free energy equation

$$\Delta G = -\Delta G_v \cdot \frac{\alpha}{2\gamma_{cv}} \cdot l^3 + \alpha l^2 + 4l \cdot \frac{\alpha}{2\gamma_{cv}} \cdot l \cdot \gamma_{cv} = -\Delta G_v \cdot \frac{\alpha}{2\gamma_{cv}} \cdot l^3 + 3\alpha l^2$$

$$\therefore \frac{d\Delta G}{dl} = -\frac{3\alpha \Delta G_v}{2\gamma_{cv}} \cdot l^2 + 6\alpha l = 0. \longrightarrow 6\alpha l = \frac{3\alpha \Delta G_v}{2\gamma_{cv}} \cdot l^2 \qquad \boxed{l = \frac{4\gamma_{cv}}{\Delta G_v}}.$$

$$l = \frac{4\gamma_{cv}}{\Delta G_v}, \quad \cancel{h = \frac{2\gamma_{cv}}{\gamma_{cv} + \gamma_{sc} - \gamma_{vs}}} \qquad \boxed{h = \frac{\alpha}{2\gamma_{cv}} \cdot l = \frac{2(\gamma_{cv} + \gamma_{sc} - \gamma_{vs})}{\Delta G_v}}.$$

Substituting l & h to Gibb's free energy gives

$$\Delta G = -\frac{32\alpha}{\Delta G_v^2} \cdot \gamma_{cv}^2 + \frac{16\alpha}{\Delta G_v^2} \gamma_{cv}^2 + \frac{32\alpha}{\Delta G_v^2} \cdot \gamma_{cv}^2 = \boxed{\frac{16\gamma_{cv}^2}{\Delta G_v^2} (\gamma_{cv} + \gamma_{sc} - \gamma_{vs})}.$$