Phase transformation #4 20192742 손수정

1. (a) Nucleation은 homogeneous 하게 일어나며 nuclei의 형태는 spherical type.

Energy barrier for spontaneous nucleation

$\Delta G = -\Delta G_v \cdot \frac{4}{3}\pi r^3 + 4\pi r^2 \gamma$

$\frac{d\Delta G}{dr} = -\Delta G_v \cdot 4\pi r^2 + 8\pi r \gamma$

$r^* = \frac{2r}{\Delta G_v}$

energy barrier는 $\Delta G = \frac{16\pi r^3}{4\Delta G_v^2}$

Nucleation은 energy가 ΔG 보다 높아야 발생함

In Maxwell-Boltman distribution

$I = I_0 \exp\left(-\frac{\Delta G}{kT}\right)$

$\ln I - \ln I_0 = -\frac{\Delta G}{kT}$ slope $-23.8\times10^3 k = -\frac{\Delta G}{k} = -\frac{1}{k}\frac{16\pi r^3}{3\Delta G_v^2}$

$\therefore \gamma = 0.28\ J/m^2$

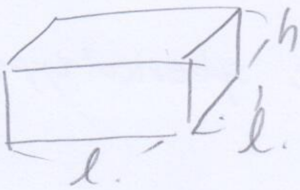
(b) critical radius $r^* = \frac{2r}{\Delta G_v} = 1.16\times10^{-9}\ m$

(c) # of Tin atoms.

tin size $1.5\times10^{-10} m$ $\quad \frac{4}{3}\pi r^{*3} = \frac{4}{3}\pi r_{tin}^3 \times n$

$1.16^3\times10^{-27} = 1.5^3\times10^{-30} \times n$

$n \fallingdotseq 462$ (개)

2. 

$V = l^2 h.$

surface energy 변화량

$$\Delta\gamma = (4hl + l^2)\gamma_{cv} + l^2\gamma_{sc} - l^2\gamma_{vs}$$

$$\Delta G = -\Delta G_v \underbrace{l^2 h}_{V} + (4hl + l^2)\gamma_{cv} + l^2\gamma_{sc} - l^2\gamma_{vs}$$

$$0 = 2lh + l^2\frac{\partial h}{\partial l} \qquad \boxed{\frac{\partial h}{\partial l} = -\frac{2h}{l}}$$

$$\frac{\partial \Delta G}{\partial l} = -2\Delta G_v lh - 2\Delta G_v l^2\frac{dh}{dl} + \left(4h + 4l\frac{dh}{dl} + 2l\right)\gamma_{cv} + 2l\gamma_{sc} - 2l\gamma_{vs}$$

위식은 critical size 에서 0

$$-2\Delta G_v lh \overset{+4}{-} \Delta G_v \cancel{l^2}\cdot\left(\frac{2h}{\cancel{l}}\right) + (4h - 8h + 2l)\gamma_{cv} + 2l\gamma_{sc} - 2l\gamma_{vs} = 0$$

$$2\Delta G_v lh + (4h + 2l)\gamma_{cv} + 2l\gamma_{sc} - 2l\gamma_{vs} = 0$$

$$l = \frac{2h\gamma_{cv}}{\gamma_{cv} + \gamma_{sc} - \gamma_{vs}} = \frac{2\gamma_{cv}}{x}\cdot h \quad (\gamma_{cv} + \gamma_{sc} - \gamma_{vs} = x.)$$

$$\Delta G = -\Delta G_v \cdot \frac{x}{2\gamma_{cv}}\cdot l^2 + xl^2 + 4l\frac{x}{2\gamma_{cv}} l\gamma_{cv} = -\Delta G_v\frac{x}{2\gamma_{cv}}\cdot l^3 + 3xl^2$$

$$\frac{d\Delta G}{dl} = 0 \qquad 6xl = \frac{3x\Delta G_v}{2\gamma_{cv}}\cdot l^2 \qquad l = \frac{4\gamma_{cv}}{\Delta G_v}$$

$$h = \frac{2(\gamma_{cv} + \gamma_{sc} - \gamma_{vs})}{\Delta G_v}$$

$$\Delta G = -\frac{32x}{\Delta G_v^2}\gamma_{cv}^2 + \frac{16x}{\Delta G_v^2}\gamma_{cv} + \frac{32x}{\Delta G_v^2}\gamma_{cv}^2 = \frac{16\gamma_{cv}^2}{\Delta G_v^2}(\gamma_{cv} + x_{sc} - \gamma_{vs})$$