Problem Set # 4 2021-20000 정광에너지과 조영근

1.

(a) Nucleation rate $I = f_0 N_0 \exp(-\Delta G^*/kT)$

$\rightarrow \ln I = \ln(f_0 N_0) - \frac{\Delta G^*}{k}\frac{1}{T}$ ... ①

Homogeneous nucleation 에서 $\Delta G^* = \frac{16}{3}\pi \frac{r^3}{(\Delta G_v)^2}$ ... ②

①: 113°C 에서 $\ln I$ vs. $1/T$ 기울기 $-\frac{\Delta G^*}{k} = -23.8 \times 10^{-3}$ [K]

$\rightarrow \Delta G^* = (23.8\times10^{-3})(1.38\times10^{-23})$

$= 3.28\times10^{-19}$ [J]

②에 ΔG^* 대입하면 $\gamma = 0.058$ J m^{-2}

(b) $r^* = \frac{2\gamma}{\Delta G_v}$

$= \frac{2(0.058)}{10^8}$

$= 1.16\times10^{-9}$ m

(c) $n^* = \frac{32\pi}{3v}\left(\frac{\gamma}{\Delta G_v}\right)^3$

atomic volume $v = \frac{4}{3}\pi(1.5\times10^{-10})^3$

$= 1.41\times10^{-29}$ m^3

$\rightarrow n^* = 465$

2.

$\Delta G = -V\Delta G_v + \Sigma A_i \gamma_i$

$= -l^2 h \Delta G_v + 4hl\gamma_{cv} + l^2\gamma_{cv} + l^2\gamma_{sc} - l^2\gamma_{vs}$ ... ①

Nuclei 가 수직이므로 계면에너지 ~~$\gamma_{cv}\cos\theta + \gamma_{sc} = \gamma_{vs}$~~ ... ②

②를 ①에 대입하면

$\Delta G = -l^2 h \Delta G_v + 4lh\gamma_{cv} + l^2\gamma_{cv}$ ... ③

$$\left.\frac{\partial \Delta G}{\partial h}\right|_{\substack{h=h^* \\ l=l^*}} = -l^* \Delta G_v + 4 l^* r_{cv} = 0 \rightarrow l^* = \frac{4 r_{cv}}{\Delta G_v}$$

$$\left.\frac{\partial \Delta G}{\partial l}\right|_{\substack{h=h^* \\ l=l^*}} = -2 l^* h^* \Delta G_v + 4 h^* r_{cv} + 2 l^* r_{cv} = 0 \rightarrow h^* = \frac{l^* r}{l^* \Delta G_v - 2 r_{cv}}$$

$$= \frac{\frac{2 \cancel{4} r_{cv}^2}{\Delta G_v}}{2 r_{cv}}$$

$$= \frac{2 r_{cv}}{\Delta G_v}$$

l^* 와 h^* 를 ③에 대입하면

$$\Delta G^* = -\left(\frac{4 r_{cv}}{\Delta G_v}\right)^2 \left(\frac{2 r_{cv}}{\Delta G_v}\right) \Delta G_v + 4 \left(\frac{4 r_{cv}}{\Delta G_v}\right)\left(\frac{2 r_{cv}}{\Delta G_v}\right) r_{cv} + \left(\frac{4 r_{cv}}{\Delta G_v}\right)^2 r_{cv}$$

$$= -\frac{32 r_{cv}^3}{(\Delta G_v)^2} + \frac{32 r_{cv}^3}{(\Delta G_v)^2} + \frac{16 r_{cv}^3}{(\Delta G_v)^2}$$

$$= \frac{16 r_{cv}^3}{(\Delta G_v)^2}$$