Problem Set #4.

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1. (a) nucleation rate of homogeneous nucleation I.

$$I = f_0 N_0 \exp\left(-\frac{\Delta G^*}{kT}\right)$$

Homogeneous nucleation 에서 r^*, ΔG^* 를 구하면

$$\Delta G = -\frac{4}{3}\pi r^3 \Delta G_v + 4\pi r^2 \gamma$$

$$\frac{\partial \Delta G}{\partial r} = 0, \quad r^* = \frac{2\gamma}{\Delta G_v}, \quad \Delta G^* = \frac{16}{3}\pi \frac{\gamma^3}{(\Delta G_v)^2}$$

ΔG^*를 위의 nucleation rate 식에 대입,

$$I = f_0 N_0 \exp\left(-\frac{16}{3}\pi \frac{\gamma^3}{(\Delta G_v)^2} \cdot \frac{1}{kT}\right) \xrightarrow{\ln} \ln I = \ln f_0 N_0 - \left(\frac{16}{3}\pi \frac{\gamma^3}{(\Delta G_v)^2 k}\right) \cdot \frac{1}{T}$$

$\ln I$와 $\frac{1}{T}$의 관계를 이용하여 γ를 구할 수 있다.

$$-23.8 \times 10^3 k = -\frac{16}{3}\pi \frac{\gamma^3}{(-10^8 J m^{-3})^2 \cdot 1.38 \times 10^{-23} J/K}.$$

$$\underline{\gamma = 0.058 J/m^2}$$

(b). $r^* = \frac{2\gamma}{\Delta G_v}$ ((a)에서 구함)

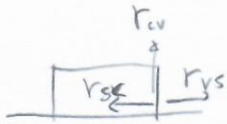
$$= \frac{2 \cdot 0.058 J/m^2}{10^8 J m^{-3}} = 1.16 \times 10^{-9} m = \underline{1.16\ nm}$$

(c) $\Delta G = -nv\Delta G_v + (36\pi)^{\frac{1}{3}} n^{\frac{2}{3}} v^{\frac{2}{3}} \gamma$ (v: molecular (atomic) volume, n: number of atoms in cluster)

$$\left.\frac{\partial \Delta G}{\partial n}\right|_{n=n^*} = 0, \quad n^* = \frac{32\pi}{3v}\left(\frac{\gamma}{\Delta G_v}\right)^3, \quad v = \frac{4}{3}\pi r^3 = 1.414 \times 10^{-29} m^3$$

$$n^* = \frac{32\pi}{3 \times 1.414 \times 10^{-29} m^3} \times \left(\frac{0.058 J/m^2}{10^8 J/m^{-3}}\right)^3 \approx 462 개$$

2. 



힘 평형 조건 : $r_{cv} = r_{vs}$

$$\Delta G = -V_{solid}\,\Delta G_v + \sum A_j r_j = -l^2 h\,\Delta G_v + \cancel{r_{sc} l^2} + r_{cv}(l^2 + 4hl) - \cancel{l^2 r_{vs}}$$

$$= -l^2 h\,\Delta G_v + r_{cv}(l^2 + 4hl)$$

critical size를 찾기 위해 ΔG를 h, l에 대해서 편미분하여 0이 되는 h^*, l^*을 찾는다

$$\frac{\partial \Delta G}{\partial h} = 0 \qquad \frac{\partial \Delta G}{\partial h} = -l^2 \Delta G_v + 4l\,r_{cv} = 0 \qquad \underline{l^* = \frac{4r_{cv}}{\Delta G_v}}$$

대입

$$\frac{\partial \Delta G}{\partial l} = 0 \qquad \frac{\partial \Delta G}{\partial l} = -2lh\,\Delta G_v + r_{cv}(2l + 4h) = 0$$

$$= -8h\,r_{cv} + r_{cv}\left(\frac{8r_{cv}}{\Delta G_v} + 4h\right) = 0, \quad 4h = \frac{8r_{cv}}{\Delta G_v} \qquad \underline{h^* = \frac{2r_{cv}}{\Delta G_v}}$$

$$\Rightarrow h = \frac{1}{2}l, \quad 2h = l$$

$$\Delta G^* = -4h^3 \Delta G_v + r_{cv}(4h^2 + 8h^2)$$

$$= -4\cdot\left(\frac{2r_{cv}}{\Delta G_v}\right)^3 \Delta G_v + r_{cv}\cdot 12\cdot\left(\frac{2r_{cv}}{\Delta G_v}\right)^2 = \underline{\frac{16}{\Delta G_v^2} r_{cv}^3}$$

$$\therefore\ l^* = \frac{4r_{cv}}{\Delta G_v}, \quad h^* = \frac{2r_{cv}}{\Delta G_v}, \quad \Delta G^* = \frac{16}{\Delta G_v^2} r_{cv}^3$$