1. (a):

$$I = f_0 N_0 \exp(-\Delta G^* / kT)$$

$$\Rightarrow \ln I = \ln f_0 N_0 - \frac{\Delta G^*}{kT}$$

$$\Rightarrow -\frac{\Delta G^*}{k} = -23.8 \times 10^3 K$$

$$\Rightarrow \Delta G^* = -1.38 \times 10^{-23} J/K \times (-23.8 \times 10^3) K = 3.28 \times 10^{-19} J$$

$$\Delta G^* = \frac{16\pi}{3} \frac{\gamma^3}{(\Delta G_v)^2} \Rightarrow \gamma^3 = \Delta G^* \times \frac{3}{16\pi} (\Delta G_v)^2 = (3.28 \times 10^{-19}) \times \frac{3}{16\pi} \times (-10^8 J m^{-3})^2$$

$$= 1.9576 \times 10^{-4}$$

$$\boxed{\therefore \gamma = 0.058 J m^{-2}}$$

(b):

$$r^* = \frac{2\gamma}{\Delta G_v} = \frac{2 \times 0.058 J m^{-2}}{10^8 J m^{-3}} = \boxed{1.16 \times 10^{-9} m}$$

(c):

$$n^* = \frac{32\pi}{3 \times \frac{4}{3}\pi r^3} \times \left(\frac{0.058 J m^{-2}}{10^8 J m^{-3}}\right)^3 = \boxed{462}$$

2.

$$\Delta G = -V_{solid}\Delta G_V + \sum_i A_i \gamma_i$$

$$= -l^2 h \Delta G_V + \gamma_{cv}(4lh + l^2) + \gamma_{sc} l^2 - \gamma_{vs} l^2 \qquad (\gamma_{vs} = \gamma_{sc} + \gamma_{cv}\cos\theta)$$

$$= -l^2 h \Delta G_V + \gamma_{cv}(4lh + l^2)$$

$$\therefore \Delta G = -l^2 h \Delta G_V + \gamma_{cv}(4lh + l^2)$$

$$\Rightarrow \frac{d\Delta G}{dl} = -2lh\Delta G_V + \gamma_{cv}(4h + 2l) = 0$$

$$\Rightarrow \frac{d\Delta G}{dh} = -l^2\Delta G_V + \gamma_{cv} 4l = 0$$

$$\Rightarrow l\Delta G_V = 4\gamma_{cv} \Rightarrow \boxed{l = \frac{4\gamma_{cv}}{\Delta G_V}}$$

$$\Rightarrow (-l\Delta G_V + 2\gamma_{cv})h + l\gamma_{cv} = 0$$

$$\Rightarrow h = \frac{-l\gamma_{cv}}{-l\Delta G_V + 2\gamma_{cv}} = \frac{-\frac{4\gamma_{cv}}{\Delta G_V} \times \gamma_{cv}}{-\frac{4\gamma_{cv}}{\Delta G_V} \times \Delta G_V + 2\gamma_{cv}} = \frac{\frac{-4(\gamma_{cv})^2}{\Delta G_V}}{-2\gamma_{cv}}$$

$$= + \frac{+2\gamma_{cv}^2}{\gamma_{cv}\cdot\Delta G_V}$$

$$\boxed{\therefore l^* = \frac{4\gamma_{cv}}{\Delta G_V}, \quad h^* = + \frac{2\gamma_{cv}^2}{\gamma_{cv}\cdot\Delta G_V}}$$

$$\Delta G^{*}_{het} = -\left(\frac{4\gamma_{cv}}{\Delta G_v}\right)^2 \frac{+2\gamma_{cv}}{\gamma_{cv}\cdot\Delta G_v} + \gamma_{cv}\left(-4\cdot\frac{4\gamma_{cv}}{\Delta G_v}\left(+\frac{2\gamma_{cv}}{\gamma_{cv}\cdot\Delta G_v}\right) + \frac{16\gamma_{cv}^2}{\Delta G_v^2}\right)$$

$$= -\frac{32\gamma_{cv}^3}{\Delta G_v^2} + \gamma_{cv}\left[\frac{32\gamma_{cv}^2}{\Delta G_v^2} + \frac{16\gamma_{cv}^2}{\Delta G_v^2}\right]$$

$$= -\frac{32\gamma_{cv}^3}{\Delta G_v^2} + \frac{48\gamma_{cv}^3}{\Delta G_v^2} = \boxed{\frac{16\gamma_{cv}^3}{\Delta G_v^2}}$$