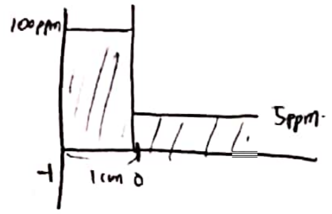
20202255 김선혁

1. (a) bottom과 edge의 ~~[illegible]~~ nitrogen losses 무시 ⇒ planar source



$$\frac{\partial C}{\partial t} = D \cdot \frac{\partial^2 C}{\partial x^2}$$

$C(x<0,\ t=0) = 100\ ppm$, $C(x>0,\ t=0) = 5\ ppm$

$C(x=0,\ t) = 5\ ppm$

(b) $C(x,t) = \frac{A}{t^{\frac{1}{2}}} \exp\left(-\frac{x^2}{4Dt}\right)$, $\int_{-\infty}^{\infty} C(x,t)\,dx = M \Rightarrow A = \frac{M}{\sqrt{4\pi D}}$

$\therefore C(x,t) = \frac{M}{\sqrt{4\pi D}} \cdot \exp\left(-\frac{x^2}{4Dt}\right)$

∘ semi-Infinite source

$$C_i = \frac{C_0\,\Delta\alpha_i}{\sqrt{4\pi Dt}} \cdot \exp\left(-\frac{(x-\alpha_i)^2}{4Dt}\right)$$

$$C(x,t) = \sum_i^{\infty} C_i = \int_{-1}^{0} \frac{100}{\sqrt{4\pi Dt}} \cdot \exp\left(-\frac{(x-\alpha_i)^2}{4Dt}\right) d\alpha + \int_0^{\infty} \frac{5}{\sqrt{4\pi Dt}} \cdot \exp\left(-\frac{(x-\alpha_i)^2}{4Dt}\right) d\alpha$$

$$= -\frac{5}{\sqrt{\pi}} \int_{x/2\sqrt{Dt}}^{-\infty} e^{-\eta^2} \cdot d\eta - \frac{100}{\sqrt{\pi}} \int_{\frac{x+1}{2\sqrt{Dt}}}^{\frac{x}{2\sqrt{Dt}}} \cdot e^{-\eta^2} d\eta$$

$\frac{x-\alpha_i}{\sqrt{4Dt}} = \eta$

$d\alpha = -2\sqrt{Dt} \cdot d\eta$

$$= \frac{100}{\sqrt{\pi}} \int_{\frac{x}{2\sqrt{Dt}}}^{\frac{x+1}{2\sqrt{Dt}}} e^{-\eta^2} d\eta + \frac{5}{\sqrt{\pi}}\left(\int_{-\infty}^{0} e^{-\eta^2} d\eta + \int_0^{\frac{x}{2\sqrt{Dt}}} e^{-\eta^2} d\eta\right)$$

$$= \frac{100}{\sqrt{\pi}}\left(\int_0^{\frac{x+1}{2\sqrt{Dt}}} e^{-\eta^2} d\eta - \int_0^{\frac{x}{2\sqrt{Dt}}} e^{-\eta^2} d\eta\right) + \frac{5}{\sqrt{\pi}}\left(\qquad '' \qquad \right)$$

$$\therefore \underset{(=\rho(x,t))}{C(x,t)} = 50\left(\mathrm{erf}\left(\frac{x+1}{2\sqrt{Dt}}\right) - \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)\right) + \frac{5}{2}\left(1 + \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)\right)$$

(c) ∘ 초기: $\int_{-1}^{0} C(x,t)\,dx = 100 \xrightarrow{t} \int_{-1}^{0} C(x,t)\,dx = 50$

$$\int_{-1}^{0} 50\left(\mathrm{erf}\left(\frac{x+1}{2\sqrt{Dt}}\right) - \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)\right) + \frac{5}{2}\left(1 + \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)\right) \cdot dx$$

// $\int_{-1}^{0} \mathrm{erf}(x)\,dx$

$= x \cdot \mathrm{erf}(x) - \frac{1}{\sqrt{\pi}}\int_{-1}^{0} e^{-x^2} \cdot 2x \cdot dx$ $\mathrm{erf}(x) \to \frac{2}{\sqrt{\pi}} e^{-x^2}$, $1 \to x$

$= x \cdot \mathrm{erf}(x) - \frac{e^{-x^2}}{\sqrt{\pi}} + C$

$$= \int_{-1}^{0} 50\,\mathrm{erf}\left(\frac{x+1}{2\sqrt{Dt}}\right) - \frac{95}{2}\left(\frac{x}{2\sqrt{Dt}}\right) + \frac{5}{2}\,dx$$

$$= 50\left[(x+1) \cdot \mathrm{erf}\left(\frac{x+1}{2\sqrt{Dt}}\right) + \frac{2\sqrt{Dt}}{\sqrt{\pi}} \cdot e^{-\frac{(x+1)^2}{4Dt}}\right]_{-1}^{0} - \frac{95}{2}\left[x \cdot \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right) + \frac{2\sqrt{Dt}}{\sqrt{\pi}} \cdot e^{-\frac{x^2}{4Dt}}\right]_{-1}^{0} + \left[\frac{5}{2}x\right]_{-1}^{0}$$

$$= 50\left(\mathrm{erf}\left(\frac{1}{2\sqrt{Dt}}\right) + \frac{2\sqrt{Dt}}{\sqrt{\pi}} e^{-\frac{1}{4Dt}} - \frac{2\sqrt{Dt}}{\sqrt{\pi}}\right) - \frac{95}{2}\left(\frac{2\sqrt{Dt}}{\sqrt{\pi}} + \mathrm{erf}\left(\frac{-1}{2\sqrt{Dt}}\right) - \frac{2\sqrt{Dt}}{\sqrt{\pi}} \cdot e^{-\frac{1}{4Dt}}\right) + \frac{5}{2}$$

$$= \frac{195}{2} \cdot \mathrm{erf}\left(\frac{1}{2\sqrt{Dt}}\right) + \frac{195}{2} \cdot \frac{2\sqrt{Dt}}{\sqrt{\pi}} \cdot e^{-\frac{1}{4Dt}} - \frac{195}{2}\frac{2\sqrt{Dt}}{\sqrt{\pi}} + \frac{5}{2}$$

$$= \frac{195}{2}\left(\mathrm{erf}\left(\frac{1}{2\sqrt{Dt}}\right) + \frac{2\sqrt{Dt}}{\sqrt{\pi}} \cdot e^{-\frac{1}{4Dt}} - \frac{2\sqrt{Dt}}{\sqrt{\pi}}\right) + \frac{5}{2} = F(t)$$

∘ $F(t) = 50 \Rightarrow t \approx 621106.111\ s = 172.53h$

(d) • $D = 4 \times 10^{-7}\,\mathrm{cm^2/s} \Rightarrow D = 8 \times 10^{-7}\,\mathrm{cm^2/s}$

$t \approx 172.53\,h$ $\qquad$ $t = 86.26\,h. \Rightarrow$ 시간이 축소됨.

• $\sqrt{Dt}$로 있다보니 $\sqrt{Dt}\uparrow \rightarrow$ 농도$\downarrow$, D가 크면 클수록 농도감소가 빨라짐 $\Rightarrow$ 50%에 도달하는 시간이 축소

2.

(c)

(a) $\Rightarrow l \propto \sqrt{t} \Rightarrow l = C\sqrt{Dt}$

(b) $\Rightarrow l^2 \propto e^{\frac{1}{T}} \Rightarrow D = D_0 \cdot \exp\left(-\frac{Q}{RT}\right)$

◦ $l = C\sqrt{D_0 \cdot \exp\left(-\frac{Q}{RT}\right) \cdot t}$
$(t = 60s)$

$\Rightarrow 2.61\,\mu m$ @ $T = 1173K \Rightarrow 2.61 \times 10^{-6} = C\sqrt{D_0 \exp(-Q/8.314 \times 1173) \cdot 60}$

~~4.42 μm @ T=~~

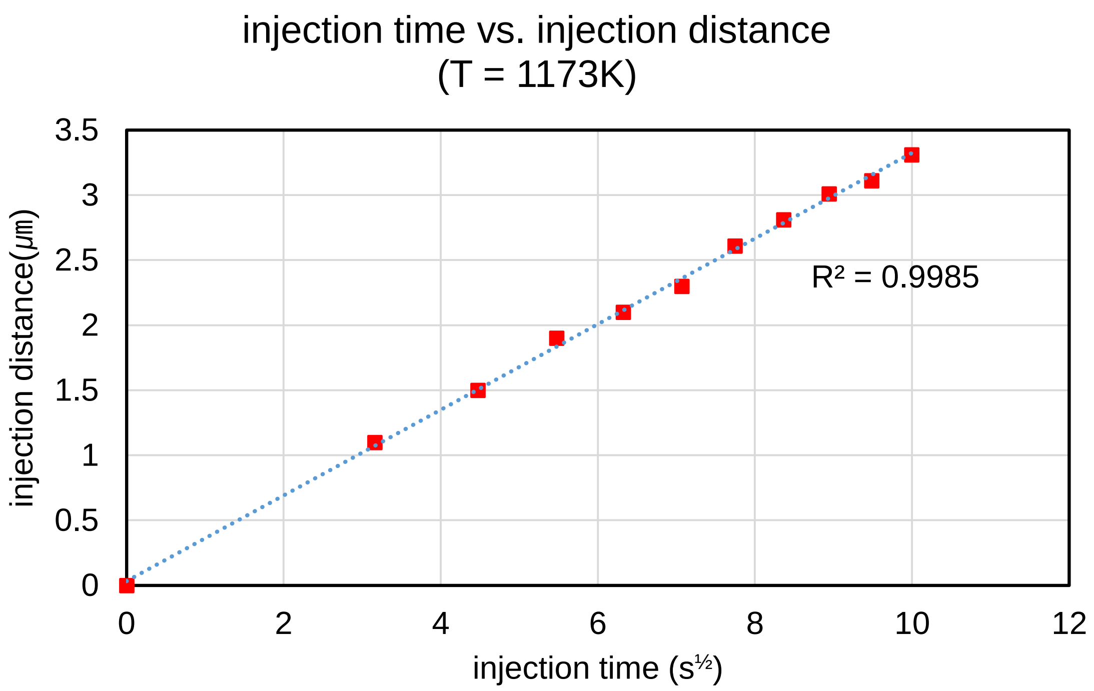
$4.61\,\mu m$ @ $T = 1273K \Rightarrow 4.61 \times 10^{-6} = C\sqrt{D_0 \cdot \exp(-Q/8.314 \times 1273) \cdot 60}$

$$\therefore \frac{2.61}{4.61} = \sqrt{\exp\left(-\frac{Q}{8.314 \times 1173} + \frac{Q}{8.314 \times 1273}\right)}$$

$Q \approx 141249\,J$

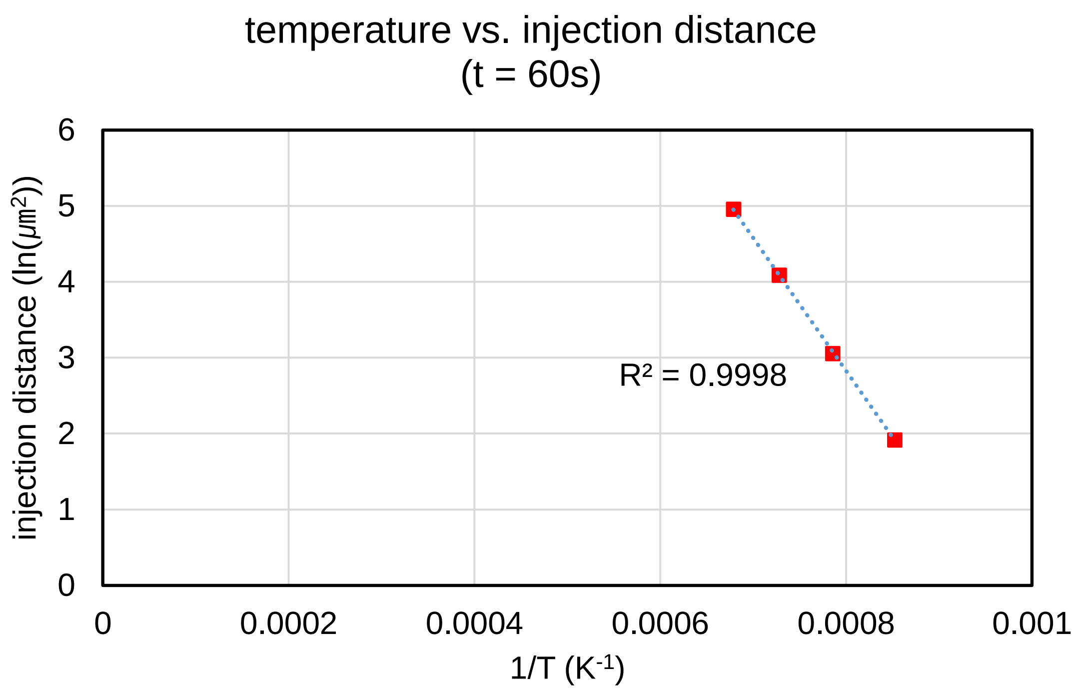
2.

(a)



$l \propto \sqrt{t}$ 의 관계를 보임

(b)



$\ln(l^2) \propto \frac{1}{T}$ 의 관계를 보임