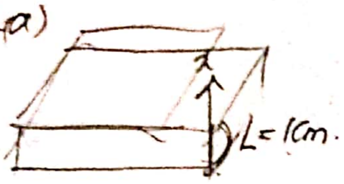
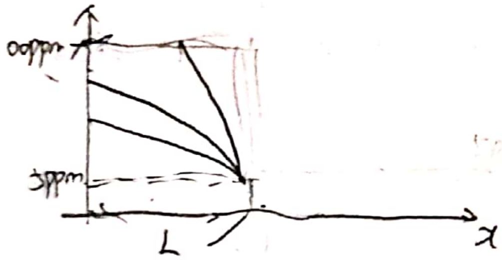
상변태론 HW 3. 20170330 이현준

1-(a)



plate의 surface 에 수직인 방향, 즉 질소가 penetrate 하는 방향의 축을 x라 하자.

그렇다면 질소의 concentration profile은 아래와 같다.



$$\frac{\partial \rho}{\partial t} = D\frac{\partial^2 \rho}{\partial t^2}$$ (by Fick's 2nd law)

boundary condition.

$\rho\,(0<x<L, t=0) = 100ppm$

$\rho\,(x=L,\ t\ ) = 5ppm$

$\frac{\partial \rho\,(x=0)}{\partial t} = 0$

(b) $\frac{\partial \rho}{\partial t} = D\frac{\partial^2 \rho}{\partial t^2}$. $\rho(x,t) = A(x)B(t)$ form.

$B(t) = \exp(-\lambda^2 Dt)$ $A(x) = A\cos\lambda x + B\sin\lambda x$.

$\rho(x,t) = \sum_n (A\cos\lambda_n x + B\sin\lambda_n x)\cdot\exp(-\lambda^2 Dt)$

$\rightarrow$ $\rho(x,t) = (A\cos\lambda_n x + B\sin\lambda_n x)\cdot\exp(-\lambda^2 Dt)$

first term approximation

$\rho\,(x=L, t) = 5ppm \Rightarrow B=0$ $\therefore \rho(x,t) = A\cos\lambda_n x\cdot\exp(-\lambda_n^2 Dt) + 5$

$\rho\,(x=0) \neq 0.$ $\rho(x,t) = \frac{4C_0}{\pi}\cos\frac{\pi x}{2L}\exp\left(-\frac{\pi^2}{4L^2}Dt\right) + 5$

(c)

$\rho(x,t) \rightarrow \frac{\rho_0}{2}$

$\frac{\rho_0}{2} = \frac{4C_0}{\pi}\int_0^1 \cos\frac{\pi}{2}x\,\exp(-\pi^2 Dt)\,dx + 5$

$\int_0^1 \cos\frac{\pi x}{2}\exp(-\pi^2 Dt)\,dx = \frac{\pi}{8} - 5$

$\Leftrightarrow \frac{2}{\pi}\exp(-\pi^2 Dt) = \frac{\pi}{8} - 5$

$t = \ln\left\{\left(5 - \frac{\pi}{8}\right)\frac{\pi}{2}\right\} \times \frac{1}{\pi^2 D}$

$t \approx 543000\ s.$

(d) 시간이 지날수록 concentration이 감소하기 때문에 D 역시 감소한다. 그렇게 되면 half concentration 될때까지 걸리는 시간이 증가한다.

2. at 1173K. Injection time vs injection distance

injection time (hr)	injection distance (Åm)
0	0
2.4	31
4.8	42
7.2	52
9.5	61
11.9	68

$\hookrightarrow$ distance $\propto \sqrt{\text{time}}$

$\Rightarrow t \propto l^2$

2 - (b)

at injection time 2.4hrs.

→ T vs distance

Temperature (K)	distance (μm)
1173	31
1273	56
1373	93
1473	143

↳ $T \propto \sqrt{Q}$

2 - (c)

$l \propto \sqrt{Dt}$ $\quad l = \alpha\sqrt{Dt}$ (α: 비례상수)

$D = D_0 \exp\left(\frac{-Q}{RT}\right)$ (Q: activation energy /mol)

과제의 데는 온도(T_1, T_2)에서 injection distance l_1, l_2를

비교하여 측정하면 Q를 구할수있다.

t=2.4hr

$T_1 = 1173K \rightarrow l = 31\mu m$ $\quad 31\times10^{-6}m = \alpha\sqrt{D_0 \exp\left(\frac{-Q}{1173R}\right) \times 2.4\times3600}$ -

$T_2 = 1473K \rightarrow l = 143\mu m$ $\quad 143\times10^{-6}m = \alpha\sqrt{D_0 \exp\left(\frac{-Q}{1473R}\right) \times 2.4\times3600}$ -

$\frac{①}{②}$ ⇒ only Q 에대한 식 $\quad Q = 146778 \approx 147723\ J$