HW 3.

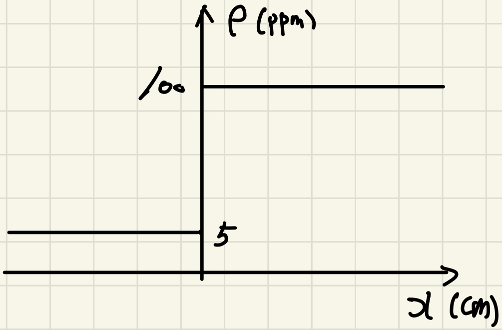
1. Iron plate : $4m \times 1m \times 1cm$

$x = 1cm$, $T = 1000K$, $D_N = 4 \times 10^{-7} cm^2 s^{-1}$

(a) bottom & edge Nitrogen loss X.

x축 diffusion 만 고려

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot J = \nabla (D \nabla \rho) = D \nabla^2 \rho = D \frac{\partial^2 \rho}{\partial x^2}$$



Boundary condition. $\rho(x,t)$:

$$\begin{cases} \rho(x>0, 0) = 100 \\ \rho(x \leq 0, t) = 5 \\ \rho(0, t) = 5 \end{cases}$$

$0 \leq x < 1 cm$

$t \geq 0$

(b) Semi-infinite condition

$$\rho_i = \frac{\rho_0}{\sqrt{4\pi D t}} \exp\left(-\left(\frac{x-\alpha_i}{\sqrt{4Dt}}\right)^2\right) dx$$

$$\begin{cases} \rho_0 = 100 \quad \text{for } \alpha_i > 0 \\ \rho_0 = 5 \quad \text{for } \alpha_i < 0 \end{cases}$$

$$\Rightarrow \rho(x,t) = \Sigma \rho$$

$$= \int_{-\infty}^{0} \frac{5}{\sqrt{4\pi D t}} \exp\left(-\left(\frac{x-\alpha_i}{\sqrt{4Dt}}\right)^2\right) d\alpha + \int_{0}^{1} \frac{100}{\sqrt{4\pi D t}} \exp\left(-\left(\frac{x-\alpha_i}{\sqrt{4Dt}}\right)^2\right) dx$$

$\frac{x-\alpha}{\sqrt{4Dt}} = \eta$ 라 하면, $d\eta = -\frac{1}{\sqrt{4Dt}} d\alpha$, $dx = -\sqrt{4Dt}\, d\eta$

$$\rho(x,t) = -\frac{5}{\sqrt{\pi}} \int_{\infty}^{\frac{x}{\sqrt{4Dt}}} e^{-\eta^2} d\eta - \frac{100}{\sqrt{\pi}} \int_{\frac{x}{\sqrt{4Dt}}}^{\frac{x-1}{\sqrt{4Dt}}} e^{-\eta^2} d\eta$$

$$= -\frac{5}{\sqrt{\pi}} \left(\int_0^{\frac{x}{\sqrt{4Dt}}} e^{-\eta^2} d\eta - \int_0^{\infty} e^{-\eta^2} d\eta \right)$$

$$- \frac{100}{\sqrt{\pi}} \left(\int_0^{\frac{x-1}{\sqrt{4Dt}}} e^{-\eta^2} d\eta - \int_0^{\frac{x}{\sqrt{4Dt}}} e^{-\eta^2} d\eta \right)$$

$$= \frac{5}{2} \left(1 - \frac{2}{\sqrt{\pi}} \int_0^{\frac{x}{\sqrt{4Dt}}} e^{-\eta^2} d\eta \right) + 50 \left(\frac{2}{\sqrt{\pi}} \int_0^{\frac{x}{\sqrt{4Dt}}} e^{-\eta^2} d\eta - \frac{2}{\sqrt{\pi}} \int_0^{\frac{x-1}{\sqrt{4Dt}}} e^{-\eta^2} d\eta \right)$$

$$= \frac{5}{2} - \frac{5}{2} \operatorname{erf}\left(\frac{x}{\sqrt{4Dt}}\right) + 50 \operatorname{erf}\left(\frac{x}{\sqrt{4Dt}}\right) - 50 \operatorname{erf}\left(\frac{x-1}{\sqrt{4Dt}}\right)$$

$$\left(\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-\eta^2} d\eta \right)$$

(c) $\int_0^1 \rho(x,t)\, dx$

$$= \int_0^1 \left(-\frac{5}{2} + \frac{5}{2} \operatorname{erf}\left(\frac{x}{\sqrt{4Dt}}\right) - 50 \operatorname{erf}\left(\frac{x}{\sqrt{4Dt}}\right) + 50 \operatorname{erf}\left(\frac{x-1}{\sqrt{4Dt}}\right) \right) dx$$

$$\left(\int \operatorname{erf}(x)\, dx = x \operatorname{erf}(x) + \frac{e^{-x^2}}{\sqrt{\pi}} + C \right)$$

$y = \frac{x}{\sqrt{4Dt}}$, $dy = \frac{1}{\sqrt{4Dt}} dx$,

$$\frac{1}{\sqrt{4Dt}} \int \operatorname{erf}\left(\frac{x}{\sqrt{4Dt}}\right) dx = \frac{x}{\sqrt{4Dt}} \operatorname{erf}\left(\frac{x}{\sqrt{4Dt}}\right) + \frac{\exp\left(-\frac{x^2}{4Dt}\right)}{\sqrt{\pi}} + C$$

$$\left(\int \operatorname{erf}\left(\frac{x}{\sqrt{4Dt}}\right) dx = x \operatorname{erf}\left(\frac{x}{\sqrt{4Dt}}\right) + \sqrt{\frac{4Dt}{\pi}} \exp\left(-\frac{x^2}{4Dt}\right) + C \right)$$

$$\int_0^1 \rho(x,t)\,dx = \int_0^1 \left(\frac{5}{2} - \frac{5}{2}\,\mathrm{erf}\left(\frac{x}{\sqrt{4Dt}}\right) + 50\,\mathrm{erf}\left(\frac{x}{\sqrt{4Dt}}\right) - 50\,\mathrm{erf}\left(\frac{x-1}{\sqrt{4Dt}}\right)\right) dx$$

$$= \frac{5}{2} + \frac{95}{2}\left[x\,\mathrm{erf}\left(\frac{x}{\sqrt{4Dt}}\right) + \frac{\sqrt{4Dt}}{\sqrt{\pi}}\exp\left(-\frac{x^2}{4Dt}\right)\right]_0^1$$

$$- 50\left[(x-1)\,\mathrm{erf}\left(\frac{x-1}{\sqrt{4Dt}}\right) + \sqrt{\frac{4Dt}{\pi}}\exp\left(-\frac{(x-1)^2}{4Dt}\right)\right]_0^1$$

$$= \frac{5}{2} - \frac{195}{2}\left(\sqrt{\frac{4Dt}{\pi}} - \sqrt{\frac{4Dt}{\pi}}\,e^{-\frac{1}{4Dt}}\right) + \frac{95}{2}\,\mathrm{erf}\left(\frac{1}{\sqrt{4Dt}}\right) - 50\,\mathrm{erf}\left(-\frac{1}{\sqrt{4Dt}}\right)$$

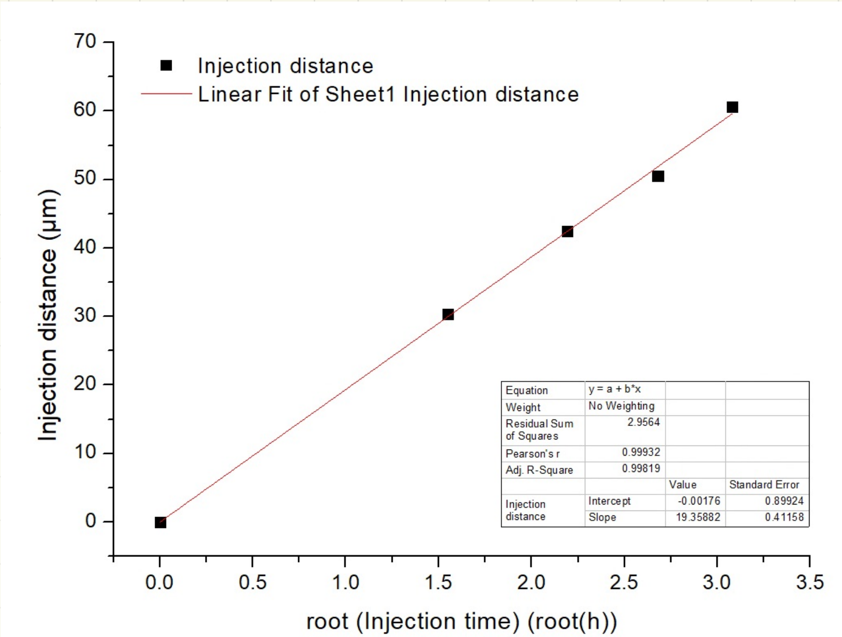
$$= \frac{5}{2} - \frac{195}{2}\left(\sqrt{\frac{4Dt}{\pi}} - \frac{\sqrt{4Dt}}{\sqrt{\pi}}\,e^{-\frac{1}{4Dt}} - \mathrm{erf}\left(\frac{1}{\sqrt{4Dt}}\right)\right)$$

$\int_0^1 \rho(x,t)\,dx = 50$ 인 t는 621106s.

(d) 위의 식에서 t가 증가할수록 식의 값은 감소한다.
여기서 4Dt를 보면 D가 커지면 t가 커지는 것과 같은 효과로 식의 값이 감소한다. 따라서 같은 값으로 되는 데에 D가 크면 그 만큼 t가 작아도 된다.
이에 따라 (c)의 값은 줄어든다.

2. 1173K 에서 $D = 1.19547 \times 10^{-13}\ m^2/s$

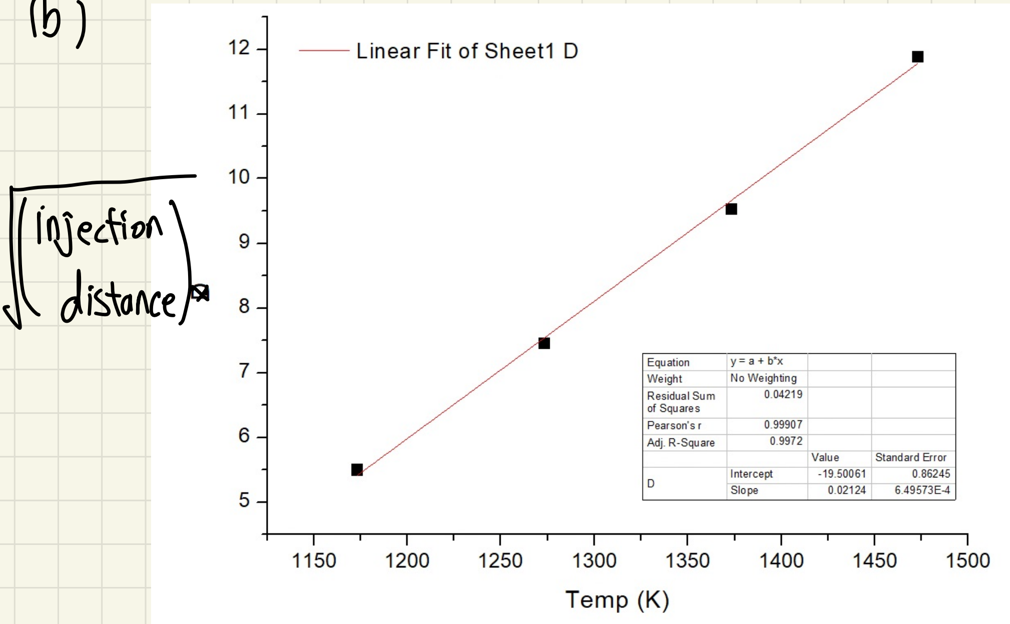
(a)



	A(X1)	B(X2)	C(Y2)
Long Name	Injection time	root(Injection time)	Injection distance
Units	h	root(h)	μm
Comments			
1	0	0	0
2	2.38	1.54272	30.3
3	4.77	2.1403	42.5
4	7.16	2.67852	50.5
5	9.54	3.08869	60.6
6			

injection distance $\propto \sqrt{\text{injection time}}$

(b)



	A(X1)	B(X2)	C(Y2)
Long Name	Temp	root(Injection time)	Injection distance afetr 24hr
Units	K	root(h)	µm
Comments			
1	1173	1.19547*10^-9	30.3
2	1273	3.92924*10^-9	55.6
3	1373	1.08592*10^-8	90.9
4	1473	2.61426*10^-8	141.4

$$\sqrt{\text{injection distance}} \propto T$$

(c) $l \propto \sqrt{t}$, $\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}$

$$l = \alpha\sqrt{Dt}$$

$$\ln l = \ln \alpha + \frac{1}{2}(\ln D + \ln t)$$

$$\ln l_1 = \ln \alpha + \frac{1}{2}(\ln D_1 + \ln t_1)$$

$$\ln l_2 = \ln \alpha + \frac{1}{2}(\ln D_2 + \ln t_2)$$

$$\ln\left(\frac{l_1}{l_2}\right) = \frac{1}{2}\left(\ln\left(\frac{D_1}{D_2}\right) + \ln\frac{t_1}{t_2}\right)$$

$\left(D = D_0 \exp\left(-\frac{Q}{RT}\right) \right)$

$t_1 = t_2$ 일 때

$$\ln\left(\frac{l_1}{l_2}\right) = \frac{1}{2}\left[\ln\left(D_0 \exp\left(-\frac{Q}{RT_1}\right)\right)\right] - \frac{1}{2}\ln\left(D_0 \exp\left(-\frac{Q}{RT_2}\right)\right)$$

$$= \frac{-Q}{2R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$T_1 = 1173K$, $T_2 = 1273K$ $t = 2.4hr$

$l_1 = 30.3\mu m$, $l_2 = 55.6\mu m$.

$$\ln\left(\frac{30.3}{55.6}\right) = -\frac{Q}{2 \times 8.314}\left(\frac{1}{1173} - \frac{1}{1273}\right)$$

$$Q = 150723J \simeq 147723J$$