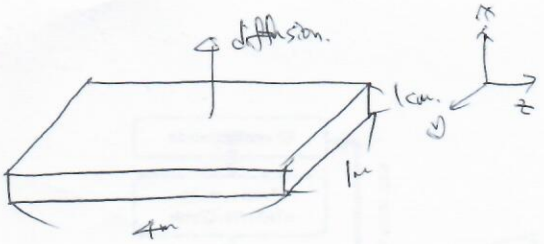
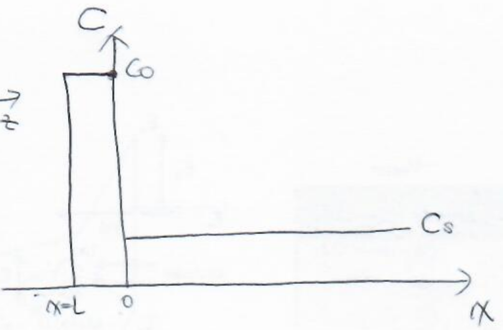
1.





(a).

$$\frac{dC}{dt} = D\cdot\frac{d^2C}{dx^2}$$

~~C(-L<x<0)~~

B.C

① $C(L<x<0,\ t=0) = C_0$

② $C(x=0,\ t) = \cancel{0}\ C_s$

③ $\dfrac{dC(x=L,t)}{dx} = 0$

(b): Separation of variable,,

$$\frac{dC(x,t)}{dt} = D\cdot\frac{d^2C}{dx^2}$$

$C(x,t) = X(x)\cdot T(t).$

$$\frac{1}{DT}\frac{dT}{dt} = \frac{1}{X}\cdot\frac{d^2X}{dx^2} = -\lambda^2$$

$\dfrac{dT}{dt} + \lambda^2 DT = 0$; $T(t) = e^{-\lambda^2 Dt}$.

$\dfrac{d^2X}{dx^2} + \lambda^2 X = 0$; $X(x) = A\cdot\cos\lambda x + B\cdot\sin\lambda x.$

$$C(x,t) = \sum_n (A\cos\lambda_n x + B\cdot\sin\lambda_n x)\cdot e^{-\lambda_n^2 Dt}$$

- initial condition.

$C(L<x<0,\ t=0) = C_0$

- boundary condition

① $C(x=0,\ t) = C_s$ ⇒ $\boxed{\text{계산이 어려워 평행이동 } (C-C_s)}$

⇒ $C(x=0,t) = 0$ → $C(x=0,t) = \sum_n A\cdot e^{-\lambda_n^2 Dt} = 0.$ ⇒ $\boxed{A=0}$

1-(a)의 B.C가 바뀜.

⇒ ① $C(L<x<0,\ t=0) = C_0 - C_s$

② $C(x=0,\ t) = 0.$

③ $\dfrac{dC(x=L,t)}{dx} = 0$

② $\dfrac{dC(x=L,t)}{dx} = 0$

⇒ $\dfrac{dC(x=L,t)}{dx} = \sum_n (B\cdot\lambda_n\cdot\cos\lambda_n\cdot x)\cdot e^{-\lambda_n^2 Dt} = 0$

⇒ $B\cdot\lambda_n\cdot\cos\lambda_n L = 0$ ⇒ $\lambda_n\cdot L = \dfrac{(2n+1)\pi}{2}$ ⇒ $\boxed{\lambda_n = \dfrac{(2n+1)\pi}{2\cdot L} \text{ or } \lambda_n = \dfrac{n\pi}{2L}\ (n=1.3.5\cdots)}$

$$\therefore\ C(x,t) = \sum_n \left(B_n \cdot \sin \frac{(2n+1)\pi}{2L} x \right) \cdot e^{-\left\{\frac{(2n+1)\pi}{2L}\right\}^2 \cdot D \cdot t}$$

$$C(x, t=0) = \sum_n^{\infty} B_n \sin\left(\frac{(2n+1)\pi}{2L} \cdot x\right) = C_0 - C_s.$$

$$B_n \int_0^L \left\{\sin \frac{(2n+1)\pi}{2L} \cdot x\right\}^2 dx = \int_0^L (C_0 - C_s) \cdot \sin \frac{(2n+1) \cdot \pi}{2L} x \, dx = B_n \cdot \frac{L}{2}$$

$$\therefore\ B_n = \frac{2}{L} \int_0^L (C_0 - C_s) \sin \frac{(2n+1) \cdot \pi}{2L} x \, dx$$

$$= \frac{2}{L}(C_0 - C_s) \cdot \int_0^L \sin \frac{(2n+1)}{2L}\pi x \cdot dx$$

$$= \frac{2}{L}(C_0 - C_s) \left[- \frac{2L}{(2n+1)\pi} \cdot \cos \frac{(2n+1)\pi x}{2L} \right]_0^L$$

$$= \frac{4}{(2n+1)\pi}(C_0 - C_s).$$

$$\boxed{\therefore\ C(x,t) = \sum_n \left\{ \frac{4}{(2n+1)\pi} (C_0 - C_s) \cdot \sin \frac{(2n+1)}{2L} \pi x \right\} e^{-\left\{\frac{(2n+1)\pi}{2L}\right\}^2 \cdot D \cdot t}}$$ //

(C).

$$\bar{C}(t) = \frac{1}{L} \int_0^L C(x,t) dx.$$

$$= \frac{1}{L}(C_0 - C_s) \sum_{n=0}^{\infty} \frac{4}{(2n+1)\pi} e^{-\left(\frac{(2n+1)\pi}{2L}\right)^2 D \cdot t} \int_0^L \sin \frac{(2n+1)\pi x}{2L} dx$$

$$= \frac{1}{L}(C_0 - C_s) \cdot \sum_{n=0}^{\infty} \frac{4}{(2n+1)\pi} e^{-\left(\frac{(2n+1)\pi}{2L}\right)^2 \cdot D \cdot t} \left[- \frac{2L}{(2n+1)\pi} \cos \frac{(2n+1)\pi}{2L} x \right]_0^L$$

$$= \boxed{\frac{8(C_0 - C_s)}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} e^{-\frac{(2n+1)^2 \pi^2}{4L^2} \cdot D \cdot t}}$$ //

C, 1st term approximation.

$$\Rightarrow \bar{C}(t) = \frac{8(C_0 - C_s)}{\pi^2} e^{-\frac{\pi^2}{4L^2} D \cdot t}.$$

$$\Rightarrow \frac{C_0 - C_s}{2} = \frac{8(C_0 - C_s)}{\pi^2} e^{-\frac{\pi^2}{4L^2} D \cdot t} \Rightarrow \frac{\pi^2}{16} = e^{-\frac{\pi^2}{4L^2} D \cdot t} \Rightarrow \ln \frac{\pi^2}{16} = -\frac{\pi^2}{4L^2} D \cdot t$$

$$\Rightarrow t = \frac{-4L^2}{\pi^2 D} \cdot \ln \frac{\pi^2}{16} = \frac{-4 \cdot (1cm)^2}{\pi^2 \cdot 4 \times 10^{-9} cm^2 \cdot s^{-1}} \ln \frac{\pi^2}{16} = \boxed{489511s}$$ //

(d): (a).(b).(c)의 경우 diffusivity가 constant한 경우 separation variable을 통해

x, t에 따른 concentration equation을 도출하였다.

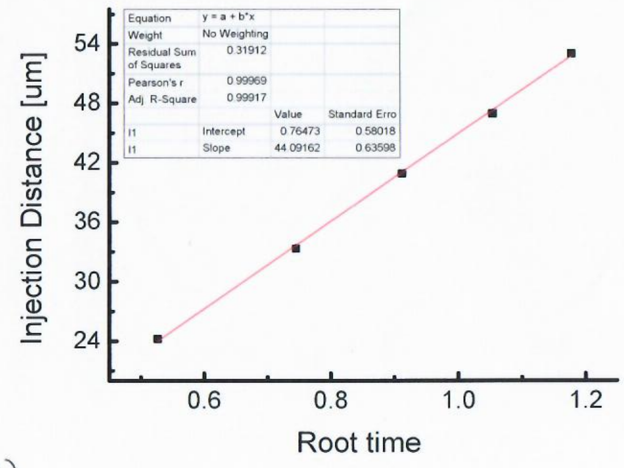
그러나 농도와 diffusivity가 비례할 경우 시간 경과에 따른 농도 감소와, 그에 따른

diffusivity가 감소할 경우, $t = \frac{-4L^2}{\pi^2 D} \ln \frac{\pi^2}{16}$ ⇒ 특정 농도 즉 50 % of initial value에

도달하기 위해 필요한 시간은 증가한다.

2

(a)



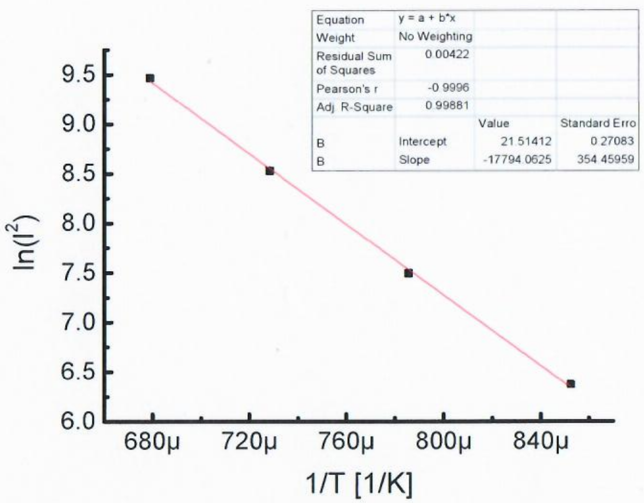
√t	Injection D
0.526519	24.24
0.744797	33.33
0.912262	40.91
1.053434	46.97
1.177804	53.03

Semi-Infinite Source.

$\Rightarrow \frac{x}{\sqrt{Dt}} = \text{constant} = A.$

$x = A\sqrt{Dt} \Rightarrow x^2 = A^2 Dt \Rightarrow x^2 \propto t \Rightarrow$ ~~...~~ $x \propto \sqrt{t}$

(b)



1/T	ln(I^2)
0.000853	6.376008
0.000786	7.49524
0.000728	8.531267
0.000679	9.466071

$\frac{x}{\sqrt{Dt}} = A$

$\Rightarrow \frac{x^2}{Dt} = A^2 \Rightarrow DtA^2 = x^2 \Rightarrow D = \frac{x^2}{tA^2} = B\exp\left(-\frac{Q}{\beta T}\right)$

$\Rightarrow \frac{x^2}{tA^2} = B\exp\left(-\frac{Q}{\beta T}\right)$

constant

$\Rightarrow \boxed{\ln x^2 \propto -\frac{Q}{\beta T}}$ //

(C):

$$D = B\exp\left(-\frac{Q}{BT}\right) = \frac{x^2}{tA^2}$$

$$\left[\begin{array}{llll} T_1 = 1173\,k & D_1 = 1.195\times10^{-9}\,cm^2/s & x_1 = 24.24\,\mu m & t = 998\,s \\ T_2 = 1273\,k & D_2 = 3.929\times10^{-9}\,cm^2/s & x_2 = 42.42\,\mu m & t = 998\,s \end{array}\right]$$

$$\left.\begin{array}{l} D_1 = B\exp\left(-\dfrac{Q}{B\cdot 1173k}\right) \\ D_2 = B\exp\left(-\dfrac{Q}{B\cdot 1273k}\right) \end{array}\right] \Rightarrow \frac{D_1}{D_2} = \exp\left(\frac{Q}{B\cdot 1273k} - \frac{Q}{1173k\cdot B}\right) = \frac{1.195\times10^{-9}}{3.929\times10^{-9}}$$

$$\Rightarrow \frac{Q}{B}\left(\frac{1}{1273} - \frac{1}{1173}\right) = \ln\frac{1.195\times10^{-9}}{3.929\times10^{-9}}$$

$$\Rightarrow \boxed{Q = \frac{B\cdot\ln\dfrac{1.195\times10^{-9}}{3.929\times10^{-9}}}{\dfrac{1}{1273} - \dfrac{1}{1173}} = 1477.64\,J}$$