



heated to 1000K
 $D = 4 \times 10^{-11} \text{ cm}^2/\text{s}$

(a) neglect Nitrogen loss from bottom & edge

\Rightarrow x-direction 1D

$$\frac{\partial p}{\partial t} = D \frac{\partial^2 p}{\partial x^2}$$

Boundary condition & initial condition

$$\left\{ \begin{array}{l} p(x < 0, t=0) = 100, \quad p(x > 0, t=0) = 5 \\ p(x=0, t) = 5 \end{array} \right\}$$

(b) Consider the case as a thin film source case

$$\Rightarrow \rho_i = \frac{\rho_0}{\sqrt{4\pi Dt}} \exp\left(-\left(\frac{x-a_i}{\sqrt{4Dt}}\right)^2\right) dx$$

$$\left. \begin{array}{l} \rho_0 = 5 \quad (a_i < 0) \\ \rho_0 = 100 \quad (a_i > 0) \end{array} \right\} \Rightarrow \rho(x, t) = \sum_i^{\infty} \rho_i$$

$$= \int_0^{\infty} \frac{5}{\sqrt{4\pi Dt}} \exp\left(-\frac{(x-a_i)^2}{4Dt}\right) da$$

$$+ \int_{-1}^0 \frac{100}{\sqrt{4\pi Dt}} \exp\left(-\frac{(x-a_i)^2}{4Dt}\right) da$$

$$\left[z = \frac{x-a}{\sqrt{4Dt}} \right] \rightarrow$$

$$e(x,t) = -\frac{5}{\sqrt{\pi}} \int_{\frac{x}{2\sqrt{Dt}}}^{-\infty} e^{-z^2} dz$$

$$- \frac{100}{\sqrt{\pi}} \int_{\frac{x+1}{2\sqrt{Dt}}}^{\frac{x}{2\sqrt{Dt}}} e^{-z^2} dz$$

$$\Rightarrow \frac{5}{\sqrt{\pi}} \left(\int_{-\infty}^0 e^{-z^2} dz + \int_0^{\frac{x}{2\sqrt{Dt}}} e^{-z^2} dz \right)$$

$$+ \frac{100}{\sqrt{\pi}} \int_{\frac{x+1}{2\sqrt{Dt}}}^{\frac{x}{2\sqrt{Dt}}} e^{-z^2} dz$$

$$\Rightarrow \frac{5}{2} \left(\operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right) + 1 \right)$$

$$+ 50 \left(\operatorname{erf}\left(\frac{x+1}{2\sqrt{Dt}}\right) - \operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right) \right)$$

$$(C) \int_{-1}^0 P(x, t) dx$$

$$= \int_{-1}^0 \frac{5}{2} \left(\operatorname{erf} \left(\frac{x}{2\sqrt{Dt}} \right) + 1 \right) dx + 50 \left(\operatorname{erf} \left(\frac{x+1}{2\sqrt{Dt}} \right) - \operatorname{erf} \left(\frac{x}{2\sqrt{Dt}} \right) \right)$$

$$\left(\begin{aligned} \int \operatorname{erf}(u) dx &= x \operatorname{erf}(u) + \frac{e^{-u^2}}{\sqrt{u}} + C \\ \int \operatorname{erf} \left(\frac{x}{2\sqrt{Dt}} \right) dx &= \frac{2\sqrt{Dt}}{\sqrt{u}} \cdot e^{-\frac{x^2}{4Dt}} \\ &\quad + x \operatorname{erf} \left(\frac{x}{2\sqrt{Dt}} \right) + C \\ \int \operatorname{erf} \left(\frac{x+1}{2\sqrt{Dt}} \right) dx &= \frac{2\sqrt{Dt}}{\sqrt{u}} \exp \left(-\frac{(x+1)^2}{4Dt} \right) \\ &\quad + (x+1) \operatorname{erf} \left(\frac{x+1}{2\sqrt{Dt}} \right) + C \end{aligned} \right.$$

$$\Rightarrow \int_{-1}^0 p(x,t) dx = \int_{-1}^0 \left(\frac{5}{2} - \frac{95}{2} \operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right) + 50 \operatorname{erf}\left(\frac{x+1}{2\sqrt{Dt}}\right) \right) dx$$

$$\Rightarrow \frac{5}{2} - \frac{195}{2} \left(\frac{2\sqrt{Dt}}{\sqrt{\pi}} - \frac{2\sqrt{Dt}}{\sqrt{\pi}} \exp\left(-\frac{1}{4Dt}\right) - \operatorname{erf}\left(\frac{1}{2\sqrt{Dt}}\right) \right)$$

when $t=0 \Rightarrow \operatorname{erf}(\infty) = 1$

$$\left[\int_{-1}^0 p(x,t) dx = 100 \right]$$

$$\Rightarrow \int_{-1}^0 p(x,t) dx = 50$$

$$t \Rightarrow 621106 \bar{z}$$

(d) As D increases, $\int_{-1}^0 P(x, t) dx$ decreases. thus, the time it takes to be 50% is reduced.

2. (a)

At 1173 K temperature

=> Injection time (h): injection distance (mm)

0	0
2.4	70
4.8	100
7.2	120
9.5	140
11.9	155
14.3	170

=> $l \propto \sqrt{t}$

(b) At 2.4 h

Temperature

1173 K

1273 K

1373 K

1473 K

Injection distance

170 μm

126 μm

210 μm

338 μm

$$\Rightarrow T \propto \sqrt{l}$$

$$(C) \quad l \propto \sqrt{t}$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}$$

→ Injection distance

$$\sqrt{Dt} \text{ or } \frac{x}{\sqrt{Dt}}$$

$$\Rightarrow l = k \sqrt{Dt}$$

$$\uparrow D_0 \exp\left(-\frac{Q}{RT}\right) \text{ activation } E$$

$$\left\{ \begin{array}{l} t = 2.4 \text{ h} \quad T = 1173 \text{ K} \\ t = 2.4 \text{ h} \quad T = 1273 \text{ K} \end{array} \right.$$

$$110 \times 10^{-6} = k \sqrt{D_0 \exp\left(-\frac{Q}{R \times 1173 \text{ K}}\right)}$$

$$126 \times 10^{-6} = k \sqrt{D_0 \exp\left(-\frac{Q}{R \times 1200 K}\right)}$$

$$\Rightarrow \underline{Q = 145944 \text{ J} \approx 146000 \text{ J}}$$