

CSL (coincidence site lattice) 현상과 관련해서 grain boundary란?

grain과 grain간의 interface를 의미한다. 일반 적으로 다른 두 grain 방향으로 인해 misorientation이 발생한다. 이는 높은 energy를 가지고 있고 2D-defect로 취급된다. 하지만 특정 misorientation angle에서 일부 lattice points들이 일치하는 경우가 발생하는데 이를 coincidence site lattice. (CSL)라고 한다. 이럴 경우 다른 misorientation angle에 비해 grain boundary energy가 매우 낮아진다. twin boundary도 이에 해당된다. 이를 표현하는데 모어 조를 사용하는 데 조는 항상 풀수값을 가지면 값이 낮을수록 misorientation이 작다. 조는 perfect crystal을 의미한다.

$$\circ \frac{\chi_i^\phi}{\chi_n^\phi} = \frac{\chi_i^\theta}{\chi_n^\theta} \cdot e^{-\Delta G_i^{seg}/RT}, \quad \Delta G_i^{seg} = [G_i^\phi - G_i^\theta] - [G_n^\phi - G_n^\theta] + RT \ln \frac{\sigma_i^\phi \sigma_n^\theta}{\sigma_n^\phi \sigma_i^\theta}$$

$$(u_i^\phi = u_i^\theta) \quad = w_i^\phi \sigma_i^\phi - w_i^\theta \sigma_n^\theta + [\Delta^{xs} \bar{G}_i^\phi - \Delta^{xs} \bar{G}_n^\theta] - [\Delta^{xs} \bar{G}_i^\theta - \Delta^{xs} \bar{G}_n^\theta]$$

$$\circ \sigma w_i = [G_i^\phi - G_i^\theta] + RT \ln \frac{\sigma_i^\phi}{\sigma_i^\theta} + RT \ln \frac{\chi_i^\phi}{\chi_i^\theta}$$

$$\sigma = \frac{1}{w_i} [G_i^\phi - G_i^\theta] + \frac{1}{w_i} [\Delta^{xs} \bar{G}_i^\phi - \Delta^{xs} \bar{G}_i^\theta] + \frac{RT}{w_i} \ln (\chi_i^\phi / \chi_i^\theta)$$

$$= \dots$$

$$= \frac{1}{w_n} [G_n^\phi - G_n^\theta] + \frac{1}{w_n} [\Delta^{xs} \bar{G}_n^\phi - \Delta^{xs} \bar{G}_n^\theta] + \frac{RT}{w_n} \ln (\chi_n^\phi / \chi_n^\theta)$$

$$\circ - \left\{ \frac{1}{w_i} [G_i^\phi - G_i^\theta] - \frac{1}{w_n} [G_n^\phi - G_n^\theta] + \frac{1}{w_i} [\Delta^{xs} \bar{G}_i^\phi - \Delta^{xs} \bar{G}_i^\theta] - \frac{1}{w_n} [\Delta^{xs} \bar{G}_n^\phi - \Delta^{xs} \bar{G}_n^\theta] \right\}$$

$$= \frac{RT}{w_i} \ln (\chi_i^\phi / \chi_i^\theta) - \frac{RT}{w_n} \ln (\chi_n^\phi / \chi_n^\theta) \quad \triangleq \Delta G_i^{seg}$$

$$\circ \ln \left[\left(\frac{\chi_i^\phi}{\chi_i^\theta} \right)^{\frac{1}{w_i}} / \left(\frac{\chi_n^\phi}{\chi_n^\theta} \right)^{\frac{1}{w_n}} \right] = -\Delta G_i^{seg} / RT$$

$$\Rightarrow \frac{\chi_i^\phi}{\chi_i^\theta} = \left(\frac{\chi_n^\phi}{\chi_n^\theta} \right)^{\frac{w_i}{w_n}} \cdot e^{-\Delta G_i^{seg} / RT} \quad (w_i/w_n = 1)$$

$$\circ \chi_i^\phi = \left(\frac{\chi_n^\phi}{\chi_n^\theta} \right) \cdot \chi_i^\theta \cdot e^{-\Delta G_i^{seg} / RT} = \left(\frac{\sum_{j=1}^{n-1} \chi_j^\phi}{\sum_{j=1}^{n-1} \chi_j^\theta e^{-\Delta G_j^{seg} / RT}} \right) \cdot \chi_i^\theta \cdot e^{-\Delta G_i^{seg} / RT} \quad \left(\sum_{j=1}^{n-1} \chi_j^\phi \chi_n^\theta = \sum_{j=1}^{n-1} \chi_j^\theta \chi_n^\phi \cdot e^{-\Delta G_j^{seg} / RT} \right)$$

$$\hookrightarrow = \frac{1 - \chi_n^\phi}{\sum_{j=1}^{n-1} \chi_j^\theta e^{-\Delta G_j^{seg} / RT}} \quad \left(\sum_{j=1}^{n-1} \chi_j^\phi = (\chi_1^\phi + \chi_2^\phi + \dots + \chi_{n-1}^\phi + \chi_n^\phi) - \chi_n^\phi \right)$$

$$= \frac{(1 - \chi_n^\phi) \cdot \chi_i^\theta \cdot e^{-\Delta G_i^{seg} / RT}}{\sum_{j=1}^{n-1} \chi_j^\theta e^{-\Delta G_j^{seg} / RT}}$$

$$= \frac{\chi_i^\theta \cdot e^{-\Delta G_i^{seg} / RT} - \chi_n^\phi \cdot \chi_i^\theta \cdot e^{-\Delta G_i^{seg} / RT}}{\sum_{j=1}^{n-1} \chi_j^\theta e^{-\Delta G_j^{seg} / RT} - \chi_n^\phi \cdot \chi_i^\theta \cdot e^{-\Delta G_i^{seg} / RT}} = \frac{\chi_i^\theta \cdot e^{-\Delta G_i^{seg} / RT} - \chi_i^\theta \cdot \chi_n^\theta}{\sum_{j=1}^{n-1} \chi_j^\theta e^{-\Delta G_j^{seg} / RT} - \chi_i^\theta \cdot \chi_n^\theta}$$

$$\circ \chi_i^\phi \left(\sum_{j=1}^{n-1} \chi_j^\theta e^{-\Delta G_j^{seg} / RT} + \chi_n^\theta \right) = \chi_i^\theta \cdot e^{-\Delta G_i^{seg} / RT} \quad (\chi_n^\theta = 1 - \sum_{j=1}^{n-1} \chi_j^\theta)$$

$$\therefore \chi_i^\phi = \frac{\chi_i^\theta \cdot e^{-\Delta G_i^{seg} / RT}}{\sum_{j=1}^{n-1} \chi_j^\theta e^{-\Delta G_j^{seg} / RT} + \chi_n^\theta}$$