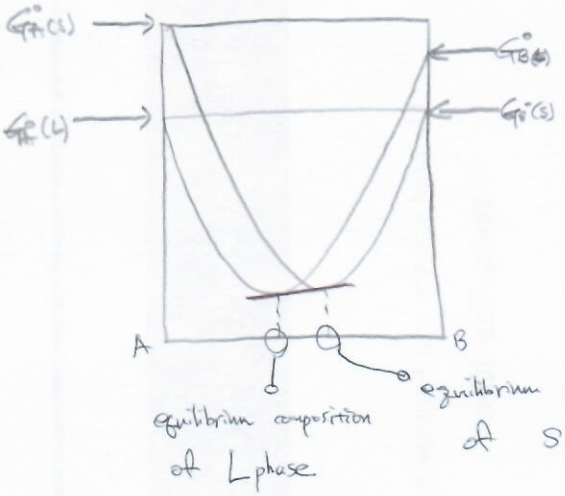
2019학기31 이승우

$$G_{(S)} = X_{A(S)} G^{\circ}_{A(S)} + X_{B(S)} G^{\circ}_{B(S)} + \Omega^{S} X_{A(S)} X_{B(S)} + RT\left(X_{A(S)} \ln X_{A(S)} + X_{B(S)} \ln X_{B(S)} \right)$$

$$G_{(L)} = X_{A(L)} G^{\circ}_{A(L)} + X_{B(L)} G^{\circ}_{B(L)} + \Omega_{L} X_{A(L)} X_{B(L)} + RT\left(X_{A(L)} \ln X_{A(L)} + X_{B(L)} \ln X_{B(L)} \right)$$



⇒ 두 상간의 상평형은 equilibrium composition에서의 G curve에서의 tangent line이 common line을 공유.

⇒ 즉 equilibrium 상태에 있는 Sphase와 Lphase에서의 A.B의 양은 다를지언정 Sphase에 있는 A와 Lphase에 있는 A의 chemical potential은 동일함.

$$\mu_{A(S)} = \mu_{A(L)}, \quad \mu_{B(S)} = \mu_{B(L)}$$

$X_A + X_B = 1 \Rightarrow X_A X_B = X_A^2 X_B + X_A X_B^2$

$$G_{(S)} = X_{A(S)} G^{\circ}_{A(S)} + X_{B(S)} G^{\circ}_{B(S)} + \Omega^{S}\left(X^2_{A(S)} X_{B(S)} + X_{A(S)} X^2_{B(S)} \right) + RT\left(X_{A(S)} \ln X_{A(S)} + X_{B(S)} \ln X_{B(S)} \right)$$

$$= X_{A(S)} \left\{ G^{\circ}_{A(S)} + \Omega_{(S)} (1 - X_{A(S)})^2 + RT \ln X_{A(S)} \right\} + X_{B(S)} \left\{ G^{\circ}_{B(S)} + \Omega_{(S)} (1 - X_{B(S)})^2 + RT \ln X_{B(S)} \right\}$$

$$\Rightarrow \boxed{\begin{aligned} \mu_{A(S)} &= G^{\circ}_{A(S)} + \Omega_{(S)} (1 - X_{A(S)})^2 + RT \ln X_{A(S)} = G^{\circ}_{A(S)} + RT \ln a_{A(S)} \\ \mu_{B(S)} &= G^{\circ}_{B(S)} + \Omega_{(S)} (1 - X_{B(S)})^2 + RT \ln X_{B(S)} = G^{\circ}_{B(S)} + RT \ln a_{B(S)} \end{aligned}}$$

$$G_{(L)} = X_{A(L)} G^{\circ}_{A(L)} + X_{B(L)} G^{\circ}_{B(L)} + \Omega_{(L)} \left(X^2_{A(L)} X_{B(L)} + X_{A(L)} X^2_{B(L)} \right) + RT \left(X_{A(L)} \ln X_{A(L)} + X_{B(L)} \ln X_{B(L)} \right)$$

$$= X_{A(L)} \left\{ G^{\circ}_{A(L)} + \Omega_{(L)} (1 - X_{A(L)})^2 + RT \ln X_{A(L)} \right\} + X_{B(L)} \left\{ G^{\circ}_{B(L)} + \Omega_{(L)} (1 - X_{B(L)})^2 + RT \ln X_{B(L)} \right\}$$

$$\Rightarrow \boxed{\begin{aligned} \mu_{A(L)} &= G^{\circ}_{A(L)} + \Omega_{(L)} (1 - X_{A(L)})^2 + RT \ln X_{A(L)} = G^{\circ}_{A(L)} + RT \ln a_{A(L)} \\ \mu_{B(L)} &= G^{\circ}_{B(L)} + \Omega_{(L)} (1 - X_{B(L)})^2 + RT \ln X_{B(L)} = G^{\circ}_{B(L)} + RT \ln a_{B(L)} \end{aligned}}$$

$\therefore \mu_{A(S)} = \mu_{A(L)} \Rightarrow G^{\circ}_{A(S)} + \Omega_{(S)} (1 - X_{A(S)})^2 + RT \ln X_{A(S)} = G^{\circ}_{A(L)} + \Omega_{(L)} (1 - X_{A(L)})^2 + RT \ln X_{A(L)}$

$$\Rightarrow G^{\circ}_{A(S)} + RT \ln a_{A(S)} = G^{\circ}_{A(L)} + RT \ln a_{A(L)}$$

$$\Rightarrow G^{\circ}_{A(L)} - G^{\circ}_{A(S)} = RT \ln a_{A(S)} - RT \ln a_{A(L)}$$

$$\Rightarrow \boxed{\Delta G_{M,A} = RT \ln \frac{a_{A(S)}}{a_{A(L)}}}$$

$\therefore \mu_{B(S)} = \mu_{B(L)} \Rightarrow \boxed{\Delta G_{M,B} = RT \ln \frac{a_{B(S)}}{a_{B(L)}}}$

∴ equilibrium S phase와 equilibrium L phase의 조성 사이의 영역에서는 minimum free energy가 common tangent line을 따라 변화하고 해당 alloy는 오직 equilibrium S phase와 equilibrium L phase의 mixture로써 구성되어짐.

따라서 각 equilibrium phase를 만들기 위한 조성은 fix되어 있으므로 각 phase의 조성비는 발생되어 있고 단지 X_A와 X_B에 따라 equilibrium S phase와 equilibrium L phase의 상대적인 amount만 변하게 된다.