상변태론 HW1 20202585 권민석

From the problem 1. $-\Delta G_{tm,A} = {}^{\circ}G_A^{\beta} - {}^{\circ}G_A^{\alpha}$. $\Delta G_{tm,\beta} = {}^{\circ}G_B^{\alpha} - {}^{\circ}G_B^{\beta}$

Using regular solution model, (binary solution $x_A + x_B = 1$)

$$\Delta G^{\alpha} = x_A\,{}^{\circ}G_A^{\alpha} + x_B\,{}^{\circ}G_B^{\alpha} + RT(x_A \ln x_A + x_B \ln x_B) + \Omega\, x_A \cdot x_B$$

$$\Delta G^{\beta} = x_A\,{}^{\circ}G_A^{\beta} + x_B\,{}^{\circ}G_B^{\beta} + RT(x_A \ln x_A + x_B \ln x_B) + \Omega\, x_A \cdot x_B$$

At phase equilibrium. $\frac{d(\Delta G^{\alpha})}{dx_B} = \frac{d(\Delta G^{\beta})}{dx_B}$ (Common tangent)

$$\frac{d\Delta G^{\alpha}}{dx_B} = -G_A^{\alpha} + G_B^{\alpha} + RT(\ln x_B^{\alpha} - \ln(1 - x_B^{\alpha}) + \Omega(1 - 2x_B^{\alpha})$$

$$= -G_A^{\alpha} + G_B^{\alpha} + RT \ln \frac{x_B^{\alpha}}{1 - x_B^{\alpha}} + \Omega(1 - 2x_B^{\alpha})$$

from same process

$$\frac{d\Delta G^{\beta}}{dx_B} = -G_A^{\alpha} + G_B^{\beta} + RT \frac{x_B^{\beta}}{1 - x_B^{\beta}} + \Omega(1 - 2x_B^{\beta})$$

$$\left.\frac{d\Delta G^{\alpha}}{dx_B}\right|_{x = x_B^{\alpha}} - \left.\frac{d\Delta G^{\beta}}{dx_B}\right|_{x = x_B^{\beta}} = -({}^{\circ}G_A^{\alpha} - {}^{\circ}G_A^{\beta}) + ({}^{\circ}G_B^{\alpha} - {}^{\circ}G_B^{\beta}) + RT \ln\left(\frac{x_B^{\alpha}}{x_B^{\beta}} \cdot \frac{1 - x_B^{\beta}}{1 - x_B^{\alpha}}\right)$$

$$+ \Omega(1 - 2x_B^{\alpha}) - \Omega(1 - 2x_B^{\beta}) = 0$$

from the condition of problem ${}^{\circ}G_A^{\alpha} - {}^{\circ}G_A^{\beta} = \Delta G_{tm,A}$, ${}^{\circ}G_B^{\alpha} - {}^{\circ}G_B^{\beta} = \Delta G_{tm,\beta}$

the equilibrium condition do not dependent on reference.

Since the reference is written in constant term, it can't effect the common tangent, which is derivative of Gibbs free energy.