

**NUMERICAL  
ANALYSIS**

8

조혜성

# 1. Code

Explicit

$dx=3*10^{-6}$  (m), 구간 개수 200개

$T= 1173, 1273, 1373, 1473$ K

$$w_{i,j+1} = w_{i,j} + \frac{\alpha \Delta t}{\Delta x^2} (w_{i+1,j} - 2w_{i,j} + w_{i-1,j})$$

U[][]에 데이터 저장  
u[i][time] vs X plot

Implicit

$dx=1*10^{-6}$  (m), 구간 개수 600개

$w^0 = [0.05, 0.01, 0.01, 0.01 \dots]$

$T= 1173, 1223, 1273, 1323, 1373, 1423, 1473$ K

$$\begin{bmatrix} (1+2\lambda) & -\lambda & 0 & \dots & 0 \\ -\lambda & & & & \\ 0 & & & & \\ \vdots & & & & \\ 0 & \dots & 0 & -\lambda & (1+2\lambda) \end{bmatrix} \begin{bmatrix} w_{1j} \\ w_{2j} \\ \vdots \\ w_{m-1,j} \end{bmatrix} = \begin{bmatrix} w_{1,j-1} \\ w_{2,j-1} \\ \vdots \\ w_{m-1,j-1} \end{bmatrix}$$

$$A w^{(j)} = w^{(j-1)}$$

$$w^j = A^{-1} w^{j-1} \text{ recursive(x5000)}$$

정해진 T에 대해  
U[]에 데이터 저장  
U[time] vs X plot

# 2. Result

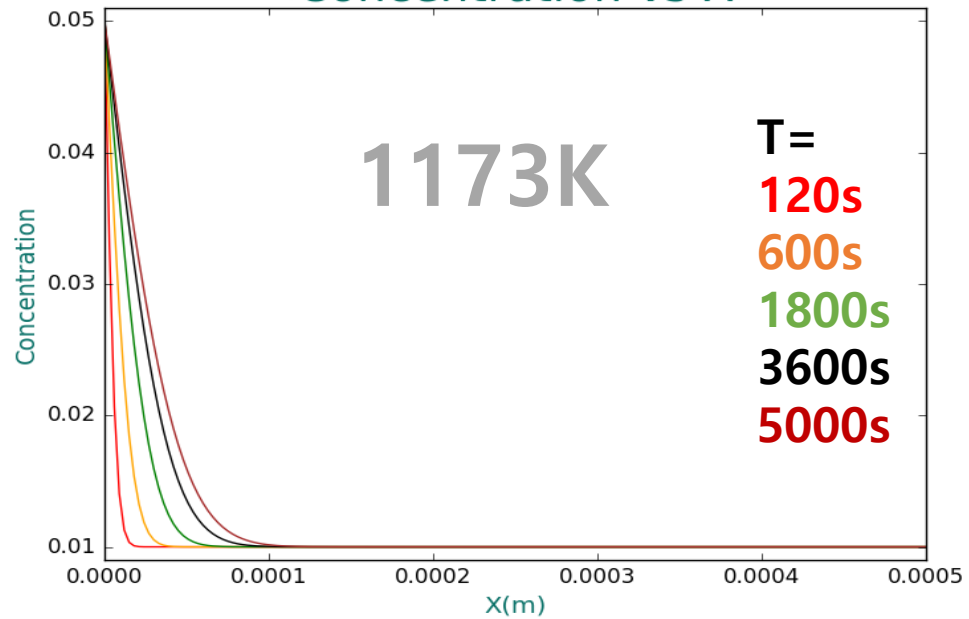
2min	10min	30min	1hour
0.05000000	0.05000000	0.05000000	0.05000000
0.01000799	0.01165476	0.01915814	0.02576909
0.01000000	0.01000445	0.01068704	0.01356820
0.01000000	0.01000000	0.01001696	0.01044465
0.01000000	0.01000000	0.01000015	0.01003038
0.01000000	0.01000000	0.01000000	0.01000116
0.01000000	0.01000000	0.01000000	0.01000003
0.01000000	0.01000000	0.01000000	0.01000000
0.01000000	0.01000000	0.01000000	0.01000000
0.01000000	0.01000000	0.01000000	0.01000000
0.01000000	0.01000000	0.01000000	0.01000000
0.01000000	0.01000000	0.01000000	0.01000000
0.01000000	0.01000000	0.01000000	0.01000000

2min	10min	30min	1hour
0.05000000	0.05000000	0.05000000	0.05000000
0.01056545	0.02001246	0.03025198	0.03553618
0.01000023	0.01090028	0.01736657	0.02389582
0.01000000	0.01002856	0.01186733	0.01635387
0.01000000	0.01000034	0.01032490	0.01241716
0.01000000	0.01000000	0.01003862	0.01076000
0.01000000	0.01000000	0.01000314	0.01019669
0.01000000	0.01000000	0.01000018	0.01004180
0.01000000	0.01000000	0.01000001	0.01000729
0.01000000	0.01000000	0.01000000	0.01000104
0.01000000	0.01000000	0.01000000	0.01000012

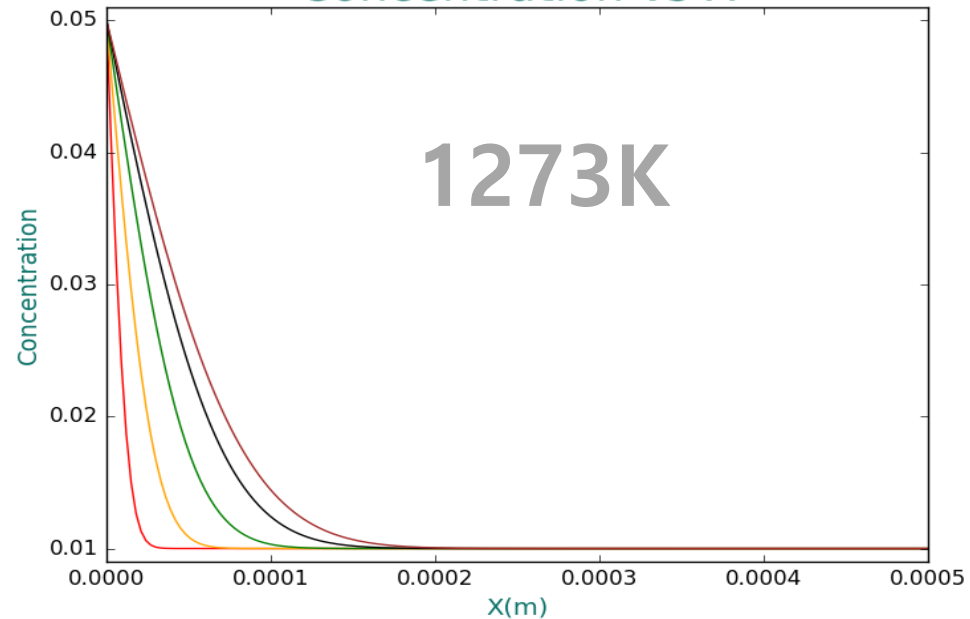
2min	10min	30min	1hour
0.05000000	0.05000000	0.05000000	0.05000000
0.01495875	0.02955193	0.03757484	0.04109826
0.01010057	0.01666337	0.02696369	0.03287596
0.01000038	0.01153064	0.01922386	0.02586081
0.01000000	0.01023307	0.01440100	0.02033218
0.01000000	0.01002340	0.01183284	0.01630730
0.01000000	0.01000155	0.01066368	0.01360040
0.01000000	0.01000007	0.01020839	0.01191850
0.01000000	0.01000000	0.01005663	0.01095294
0.01000000	0.01000000	0.01001331	0.01044073
0.01000000	0.01000000	0.01000270	0.01018962

2min	10min	30min	1hour
0.05000000	0.05000000	0.05000000	0.05000000
0.02277667	0.03622221	0.04186901	0.04421856
0.01187385	0.02489464	0.03425734	0.03862531
0.01011797	0.01723702	0.02758691	0.03339024
0.01000314	0.01297971	0.02211461	0.02864991
0.01000004	0.01103261	0.01791187	0.02449727
0.01000000	0.01029976	0.01489019	0.02097783
0.01000000	0.01007265	0.01285630	0.01809213
0.01000000	0.01001466	0.01157462	0.01580303
0.01000000	0.01000246	0.01081844	0.01404627
0.01000000	0.01000034	0.01040075	0.01274191

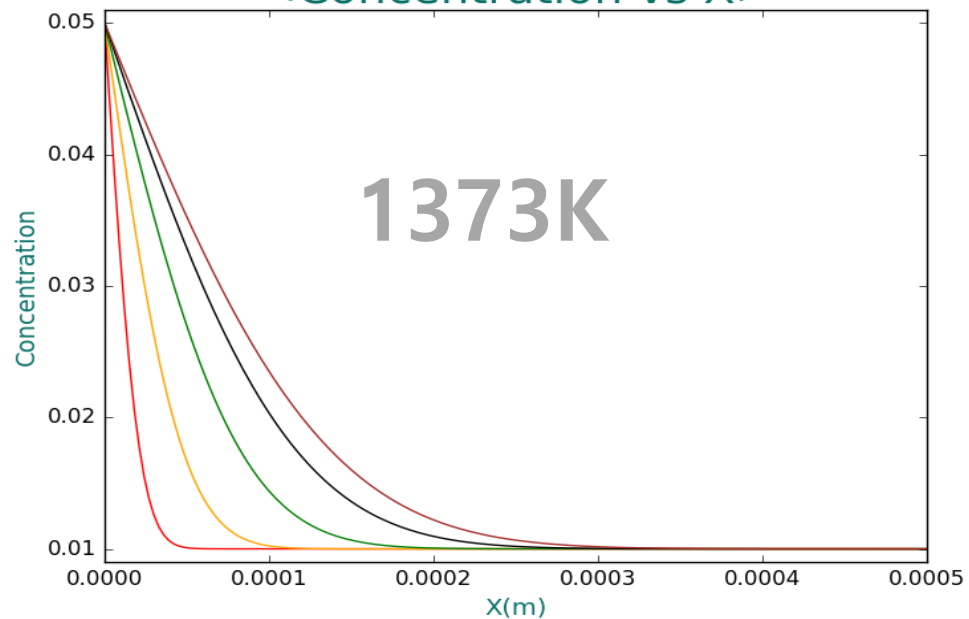
<Concentration vs X>



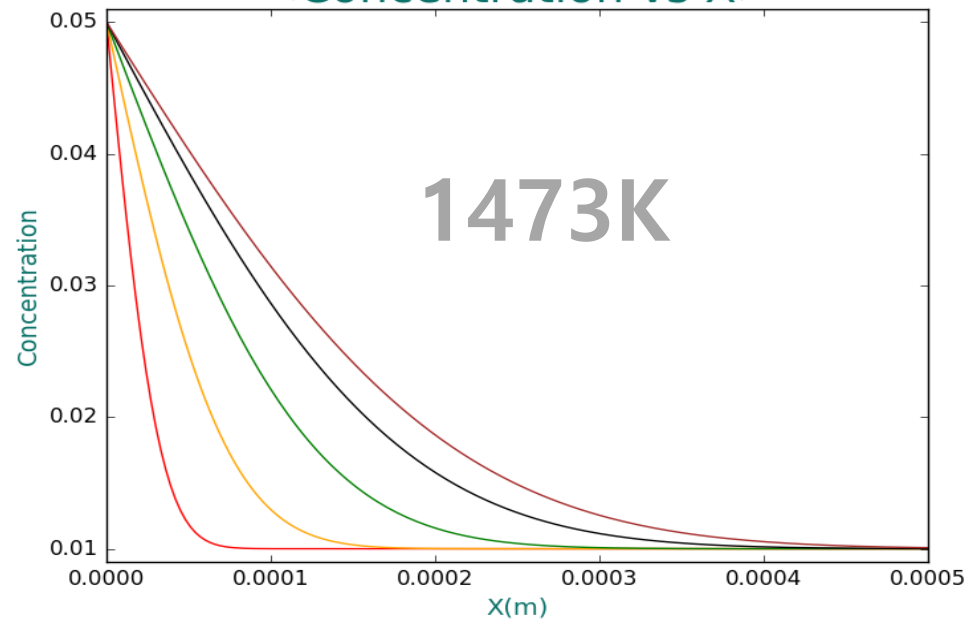
<Concentration vs X>



<Concentration vs X>

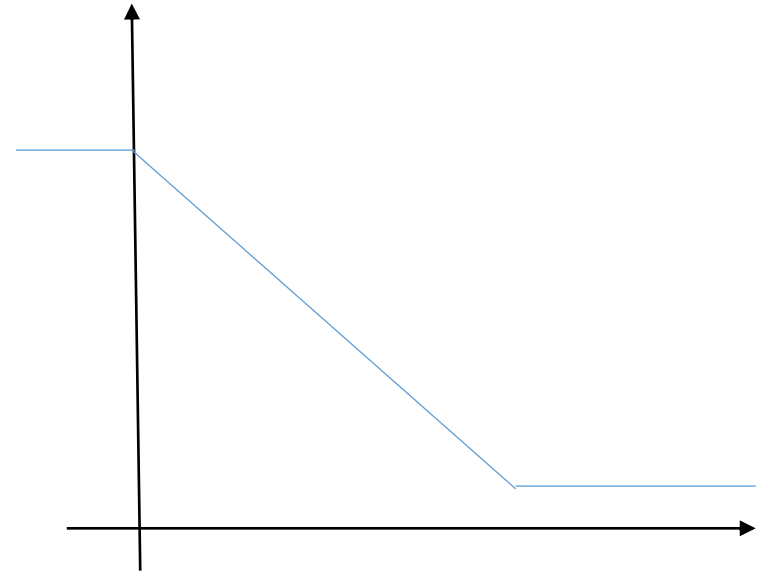
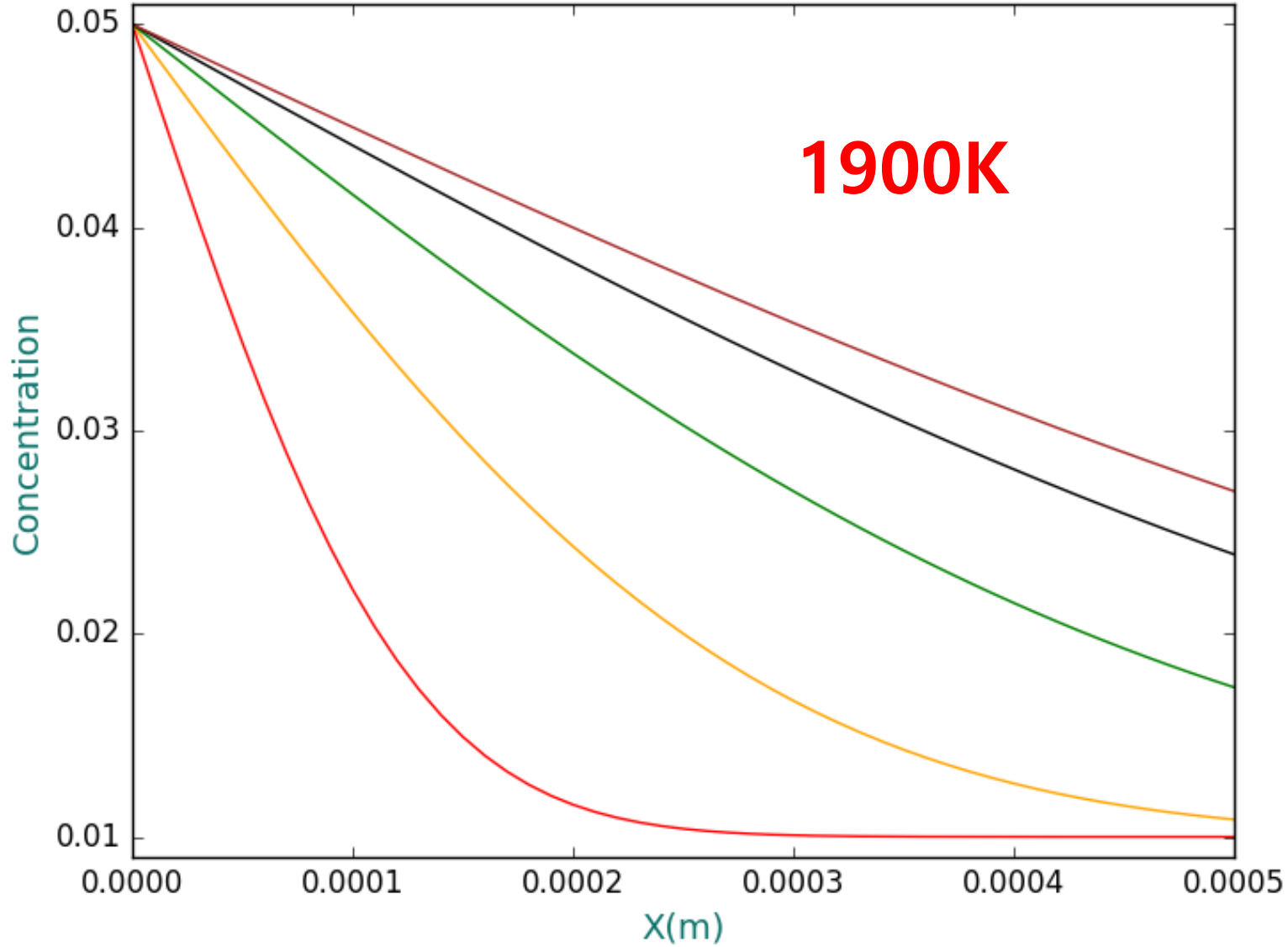


<Concentration vs X>



# <Concentration vs X>

1900K



온도가 높고, 시간이 커질수록  
위와 같은 형태에 가까워짐을  
확인 할 수 있다.

# 2. Result

Temperature = 1173

$dx = 0.0000002$

EXPLICIT METHOD

```
=====
x      2min      10min      30min      1hour
=====
0      0.05000000  0.05000000  0.05000000  0.05000000
5      -3921528068531007437891074572336047520937005260328376083029
5764491264.00000000  -inf  -inf  -inf
10     44334099806685730084589369449575931263441568081401552372742
244013568.00000000  inf  inf  inf
15     -2563858979636136960988953276044417858856409695986117337121
9525602304.00000000  -inf  -inf  -inf
20     89568099822178974745656581182963002757034638072816612137410
34002944.00000000  inf  inf  inf
25     -1983410322249019895408939285862573891777762883537762819751
935503360.00000000  -inf  -inf  -inf
30     28315535565237507245512733814676872414154697062587347900259
4361600.00000000  inf  inf  inf
35     -2616663712343746911144606565719097131906768156132138551470
3585408.00000000  -inf  -inf  -inf
40     15604254883494882954724500831734515670864035657262448031345
36736.00000000  inf  inf  inf
45     -5956658590758079764528451164126531382419962674771748378837
3008.00000000  -inf  -inf  -inf
50     14377510578770875717119097446230020362972041644092942666952
52.00000000  inf  inf  inf
=====
```

# 2. Result

Temperature = 1173  
 dx = 0.0000002

EXPLICIT METHOD

x	2min	10min	30min	1hour
0	0.05000000	0.05000000	0.05000000	0.05000000
5	-3921528068531007437891074572336047520937005260328376083029	-inf	-inf	-inf
10	5764491264.00000000	-inf	-inf	-inf
15	44334099806685730084589369449575931263441568081401552372742	inf	inf	inf
20	244013568.00000000	inf	inf	inf
25	-256385897963613696098895327604441785885640969598611733712	-inf	-inf	-inf
30	9525602304.00000000	-inf	-inf	-inf
35	89568099822178974745656581182963002757034638072816612137410	inf	inf	inf
40	34002944.00000000	inf	inf	inf
45	-1983410322249019895408939285862573891777762883537762819751	-inf	-inf	-inf
50	935503360.00000000	-inf	-inf	-inf
52	28315535565237507245512733814676872414154697062587347900259	inf	inf	inf
54	4361600.00000000	inf	inf	inf
55	-2616663712343746911144606565719097131906768156132138551470	-inf	-inf	-inf
56	3585408.00000000	-inf	-inf	-inf
57	15604254883494882954724500831734515670864035657262448031345	inf	inf	inf
58	36736.00000000	inf	inf	inf
59	-5956658590758079764528451164126531382419962674771748378837	-inf	-inf	-inf
60	3008.00000000	-inf	-inf	-inf
61	14377510578770875717119097446230020362972041644092942666952	inf	inf	inf
62	52.00000000	inf	inf	inf

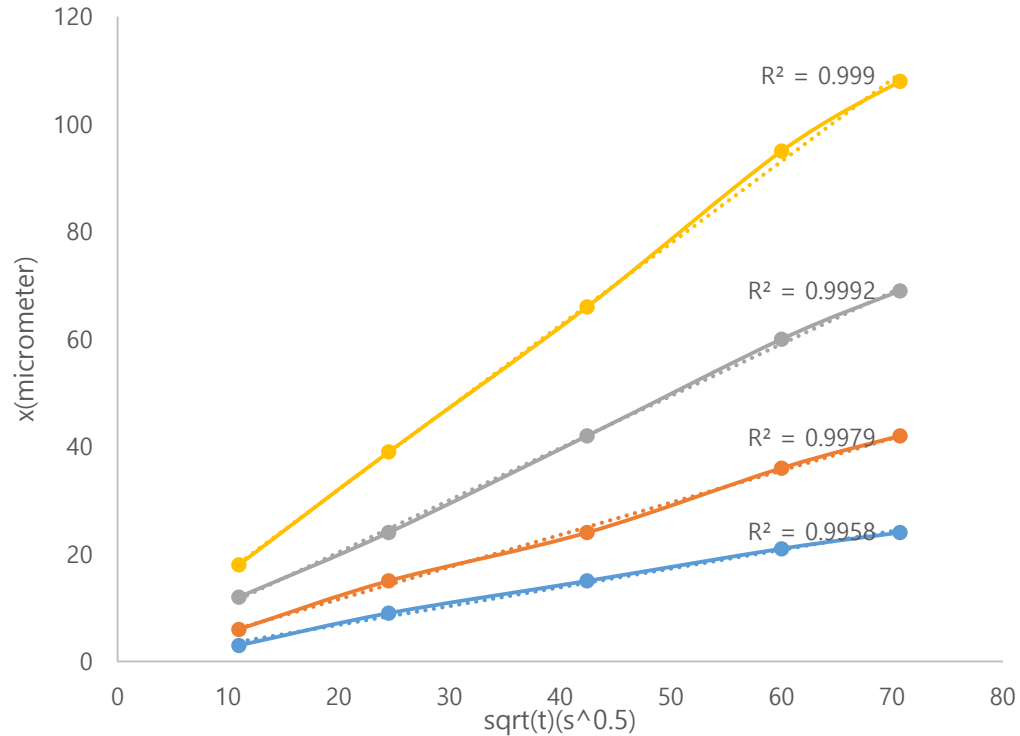
Temperature = 1173  
 dx = 0.0000002  
 IMPLICIT METHOD

x	2min	10min	30min	1hour
0	0.05000000	0.05000000	0.05000000	0.05000000
5	0.04406008	0.04733788	0.04846063	0.04885963
10	0.03832544	0.04469427	0.04692477	0.04772013
15	0.03298013	0.04208730	0.04539589	0.04658236
20	0.02816944	0.03953434	0.04387744	0.04544716
25	0.02398869	0.03705166	0.04237277	0.04431538
30	0.02047980	0.03465411	0.04088511	0.04318782
35	0.01763519	0.03235485	0.03941761	0.04206527
40	0.01540728	0.03016518	0.03797324	0.04094849
45	0.01372114	0.02809438	0.03655481	0.03983820
50	0.01248772	0.02614957	0.03516493	0.03873508

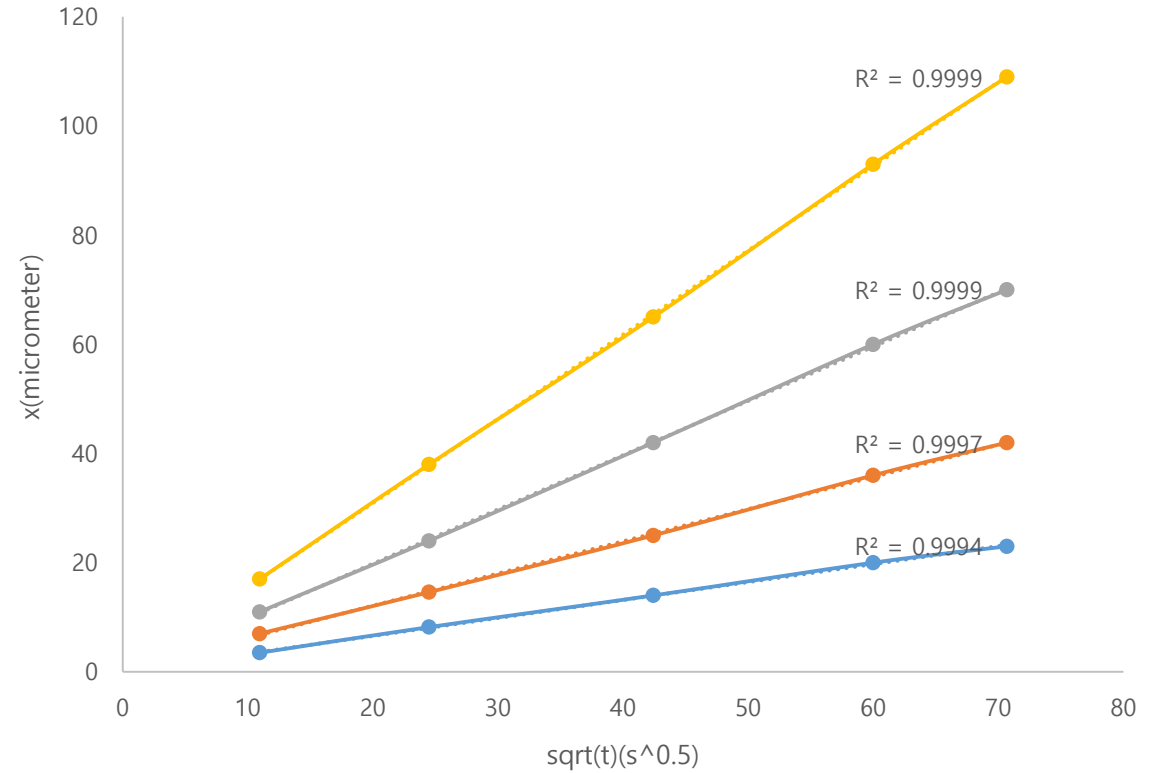
$$\frac{\alpha \Delta t}{\Delta x^2} \leq \frac{1}{2} \quad \text{Stability Condition 만족 X}$$

# 3. a(Time Dependency)

Explicit(Time dependency)



Implicit(Time dependency)

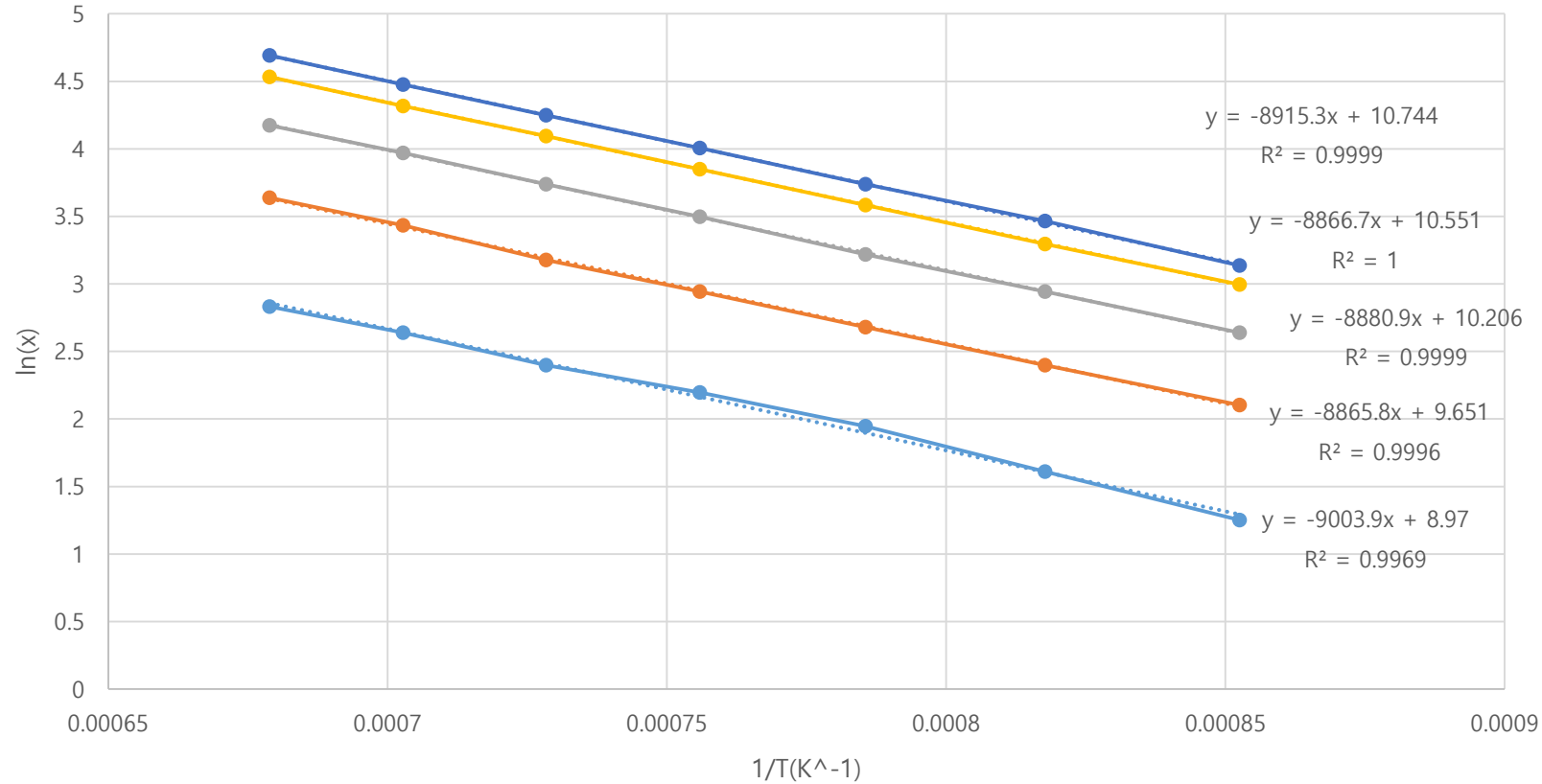


$$x = \sqrt{Dt}$$
$$x \propto \sqrt{t}$$



# 4. b(Temperature Dependency)

Implicit(Temperature Dependency)



$$x = \sqrt{Dt}$$

$$D = D_0 \exp\left(-\frac{Q}{RT}\right)$$

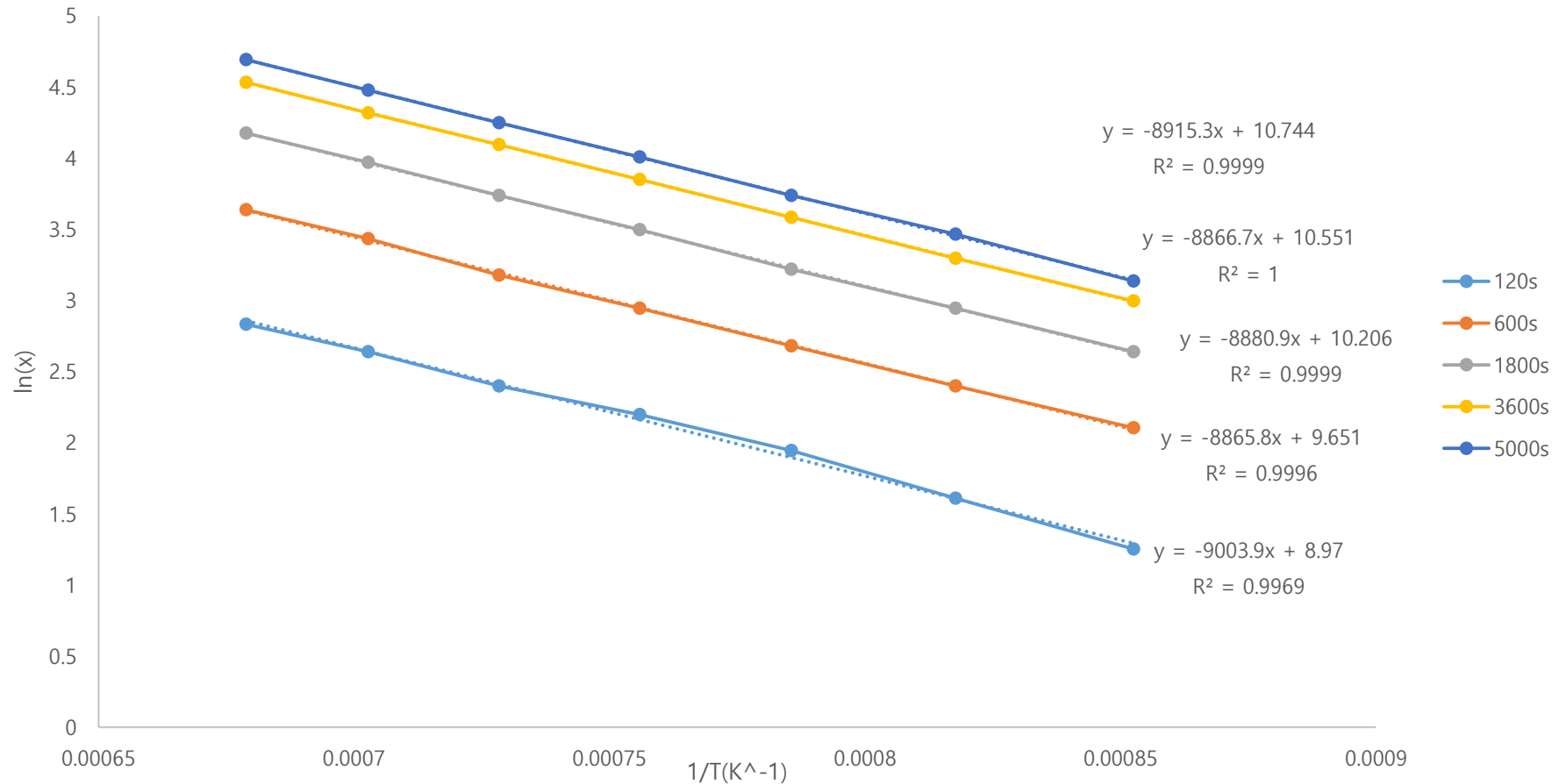
$$x \propto \exp\left(-\frac{Q}{2RT}\right)$$

$$\therefore \ln x \propto \frac{1}{T}$$

- $\ln(\text{Injection distance}) \propto \frac{1}{\text{Temperature}}$

# 4. b(Temperature Dependency)

Implicit(Temperature Dependency)



Implicit(-Q/2R)

-8915.3

-8866.7

-8880.9

-8865.8

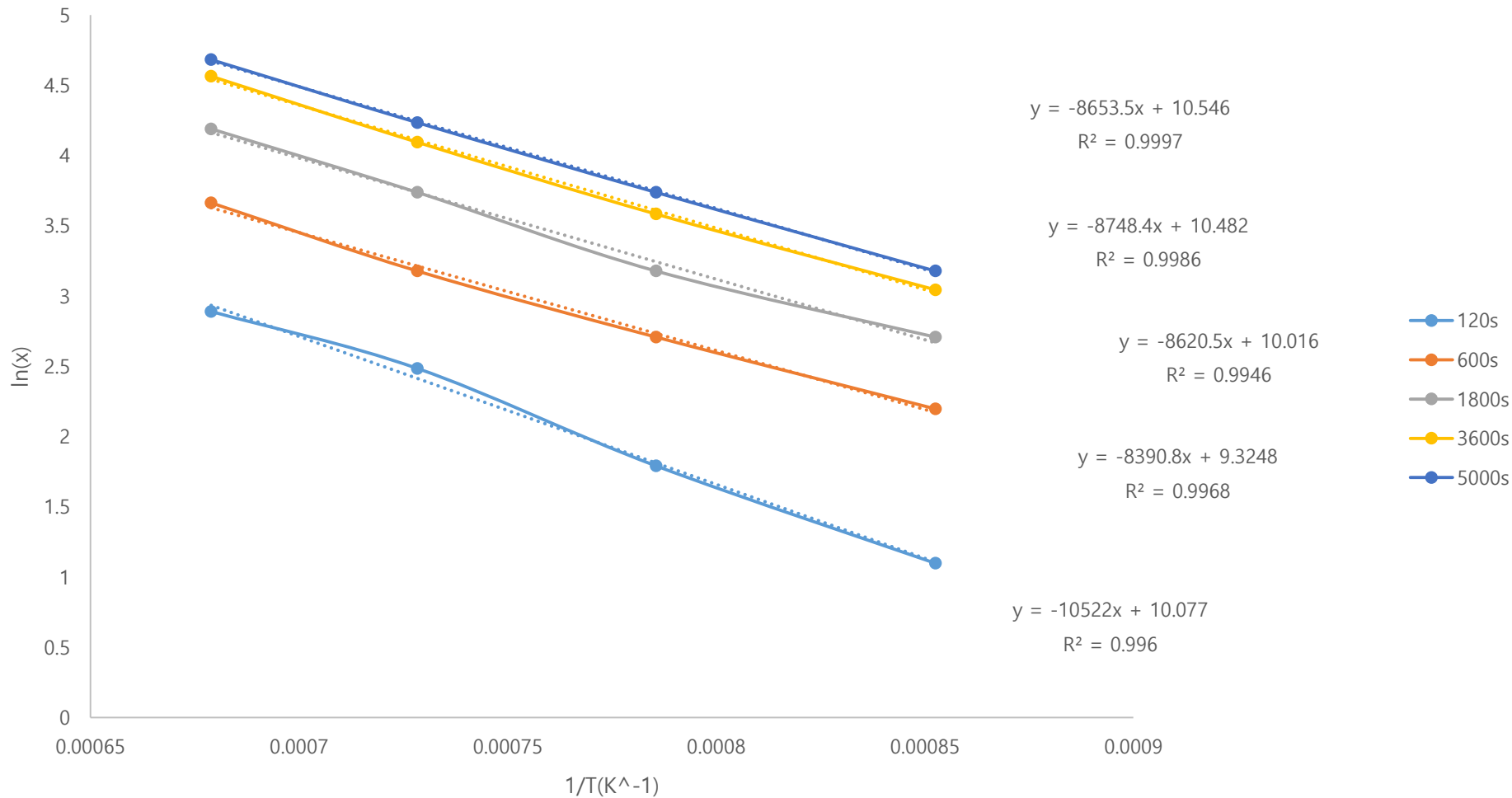
-9003.9

AVERAGE:

-8906.52

# 4. b(Temperature Dependency)

Explicit(Temperature Dependency)



**Explicit(-Q/2R)**

-8653.5

-8748.4

-8620.5

-8390.8

-10522

**AVERAGE:**

-8987.04

# 5. c-Activation Energy

Implicit(-Q/2R)	Explicit(-Q/2R)
-8915.3	-8653.5
-8866.7	-8748.4
-8880.9	-8620.5
-8865.8	-8390.8
-9003.9	-10522
AVERAGE: -8906.52	AVERAGE: -8987.04

- **Explicit 보다 implicit이 error 작게 나온다**
- **Iteration time은 explicit이 더 오래 걸린다**
- 즉, Explicit을 쓸 때에는 Implicit을 쓰자.

	Q	Relative Error(%)
Implicit	148105	0.2585
Explicit	149444	1.1650
reference	147723	

**DANKE**