

U 가 최소가 되는 조건

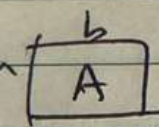
$$U = \int \gamma dl = 2 \int_0^{L_1} r_1 dl + 2 \int_0^{L_2} r_2 dl$$

$$= 2\sigma_1 L_1 + 2\sigma_2 L_2$$

$$(L_1 L_2 = A \rightarrow L_2 = \frac{A}{L_1})$$

$$\rightarrow \frac{\partial U}{\partial L_1} = 2\sigma_1 - \frac{2\sigma_2 A}{L_1^2} = 0 \rightarrow L_1 = \sqrt{\frac{\sigma_2 A}{\sigma_1}}, L_2 = \sqrt{\frac{\sigma_1 A}{\sigma_2}}$$

$$\rightarrow \frac{L_1}{L_2} = \frac{\sigma_2}{\sigma_1}$$

2. $\epsilon_i = \frac{h^2}{8m} \left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} \right)$ 

$$Z = \sum e^{-\epsilon_i / kT} = \int_0^\infty e^{-\frac{h^2}{8mkT} \left(\frac{n_x^2}{a^2} \right)} dn_x \int_0^b e^{-\frac{h^2}{8mbT} \left(\frac{n_y^2}{b^2} \right)} dn_y$$

$$= \frac{a}{2} \sqrt{\frac{8\pi mkT}{h^2}} \frac{b}{2} \sqrt{\frac{8\pi mbT}{h^2}} = A \frac{2\pi mkT}{h^2}$$

$$\rightarrow \ln Z = \ln A + \ln T + \ln \left(\frac{2\pi mb}{h^2} \right)$$

$$\rightarrow P = - \left(\frac{\partial F}{\partial A} \right)_T = \frac{NkT}{A} \rightarrow PA = NkT$$

$$U = NkT \left(\frac{\partial F}{\partial T} \right)_A = NkT \rightarrow U = NkT$$

$$3. \quad Z = \sum g_n e^{-\epsilon_n/kT} \quad \leftarrow \quad g_n = N_n C_n, \quad \epsilon_n = n\epsilon$$

$$= \left(k e^{-\frac{\epsilon}{kT}} \right)^{N+1} \quad (\because \sum_{n=0}^N n C_n = (1+2)^N)$$

$$\rightarrow \frac{\partial \ln Z}{\partial \epsilon} = \frac{1}{Z} \sum (N-n) a_n C_n e^{-n\epsilon/kT}$$

$$= Na - \frac{a}{Z} \sum_{n=0}^N n C_n e^{-n\epsilon/kT}$$

$$= Na - a kT \frac{\partial}{\partial \epsilon} (\ln Z)$$

$$\rightarrow Na - (N+1)a \left(\frac{1}{1+e^{\epsilon/kT}} \right)$$

$$6. \quad \Delta G = \Delta H - T\Delta S = 459 \times 4.2 - 298 \times (0.58 - 1.21) \times 9.2$$

$$= 2896 \text{ J}$$

$$V_g = \frac{12}{2.22} = 5.405 \text{ cm}^3/\text{mol}$$

$$V_{d2} = \frac{12}{3.55} = 3.415 \text{ cm}^3/\text{mol}$$

$$\rightarrow \Delta V = -1.99 \text{ cm}^3/\text{mol}$$

$$\frac{\partial G}{\partial p}_T = \Delta V \rightarrow \Delta G(p_1, T=298\text{K}) = \Delta G(p_2, T=298\text{K}) + \int_1^2 \Delta V dp$$

$$= 2896 + (-1.99 \times 0.1013) / (p_1) = 0$$

$$\rightarrow p = 14361 \text{ atm}$$