

2.1 An monatomic ideal gas at 300 K has a volume of 15 liters at a pressure of 15 atm. Calculate

- The final volume of the system
- The work done by the system
- The heat entering or leaving the system
- The change in the internal energy
- The change in the enthalpy when the gas undergoes
 - A reversible isothermal expansion to a pressure of 10 atm
 - A reversible adiabatic expansion to a pressure of 10 atm

The constant-volume molar heat capacity of the gas, c_v , has the value $1.5 R$.

i). reversible isothermal expansion. $\Rightarrow PV = nRT$ at 300 K.

$$a. P_1 V_1 = P_2 V_2 \Rightarrow (15 \text{ atm})(15 \text{ L}) = (10 \text{ atm}) V_2$$

$$V_2 = \frac{15 \text{ atm} \cdot 15 \text{ L}}{10 \text{ atm}} = \boxed{22.5 \text{ L}}$$

$$b. W = \int_{V_1}^{V_2} \frac{nRT}{V} dV = nRT \ln\left(\frac{V_2}{V_1}\right), P_1 V_1 = nRT, \Rightarrow n = \frac{P_1 V_1}{RT_1} = \frac{15 \cdot 15}{(0.08206)(300)}$$

$$= 9.14 \text{ mol}$$

$$\therefore W = (9.14 \text{ mol})(0.08206 \text{ atm}\cdot\text{L/mol}\cdot\text{K})(300 \text{ K}) \ln\left(\frac{22.5 \text{ L}}{15 \text{ L}}\right) = \boxed{9240 \text{ J}}$$

$$c. \text{ isothermal} \Rightarrow \Delta U = 0 \quad \therefore q - w = \Delta U = 0 \rightarrow q = w \therefore \boxed{9240 \text{ J}}$$

$$d. \Delta U = 0 (\because nC_v \Delta T = 0)$$

$$e. \Delta H = nC_p \Delta T = 0 (\because \Delta T = 0)$$

ii). reversible adiabatic expansion

$$a. P_1 V_1^\gamma = P_2 V_2^\gamma, \gamma = \frac{C_p}{C_v} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3}$$

$$(15 \text{ atm})(15 \text{ L})^{5/3} = (10 \text{ atm})(V_2)^{5/3}$$

$$(V_2)^{5/3} = \frac{15 \text{ atm}}{10 \text{ atm}} (15 \text{ L})^{5/3} \Rightarrow V_2 = \left(\frac{3}{2} (15 \text{ L})^{5/3}\right)^{3/5}$$

$$= \left(\frac{3}{2}\right)^{3/5} \cdot 15 \text{ L} = \boxed{19.1 \text{ L}}$$

$$b. \Delta U = q - w \therefore -w = \Delta U \quad w = -\Delta U = -nC_v \Delta T$$

$$= -(9.14 \text{ mol})\left(\frac{5}{2}R\right)\Delta T$$

$$T_2 = \frac{P_2 V_2}{nR} = \frac{(10 \text{ atm})(19.1 \text{ L})}{(9.14 \text{ mol})(0.08206)} = 295 \text{ K} \Rightarrow \Delta T = T_2 - T_1 = 295 \text{ K} - 300 \text{ K} = -5 \text{ K}$$

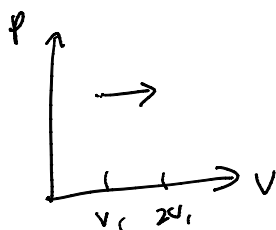
$$\therefore w = -(9.14 \text{ mol})\left(\frac{5}{2}R\right) \cdot (-5 \text{ K}) = 56.6 \text{ atm}\cdot\text{L} \quad \therefore \boxed{5660 \text{ J}}$$

$$c. \text{ adiabatic} \Rightarrow q = 0$$

$$d. \Delta U = -w = \boxed{-5660 \text{ J}}$$

$$e. \Delta H = nC_p \Delta T = (9.14 \text{ mol})\left(\frac{5}{2} \cdot 0.08206 \text{ atm}\cdot\text{L/mol}\cdot\text{K}\right)(-5 \text{ K})$$

$$= -84.3 \text{ atm}\cdot\text{L} \rightarrow \therefore \boxed{-8543 \text{ J}}$$



$$pV = nRT$$

2. 2.2 One mole of a monatomic ideal gas in the initial state $T = 273 \text{ K}$ / $P = 1 \text{ atm}$ is subjected to the following three processes, each of which is conducted reversibly:
- A doubling of its volume at constant pressure.
 - Then a doubling of its pressure at constant volume.
 - Then a return to the initial state along the path $P = 6.643 \times 10^{-4} V^2 + 0.6667$.
- Calculate the heat and work effects which occur during each of the three processes.

a. $C_p = \frac{5}{2}R$ $V_2 = 2V_1$ $T_1 = 273 \text{ K}$ $P_1 = P_2 = 1 \text{ atm}$ $n = 1 \text{ mol}$ $T_2 = \frac{P_2 V_2}{nR} = \frac{(1 \text{ atm}) \cdot (44.8 \text{ L})}{(1 \text{ mol}) (0.0820)} = 546 \text{ K}$

$P_1 V_1 = nRT_1$ $V_1 = \frac{nRT_1}{P_1} = \frac{(1 \text{ mol}) (0.0820) (273 \text{ K})}{1 \text{ atm}} = 22.4 \text{ L}$ $V_2 = 44.8 \text{ L}$ $\Delta T = 273$

$\therefore V_2 - V_1 = 22.4 \text{ L}$

$\therefore$ Constant pressure $\Rightarrow W = -\int P dV = -(1 \text{ atm})(-22.4 \text{ L}) = 22.4 \text{ atm} \cdot \text{L} \Rightarrow (22.4) \cdot (101.325 \text{ J}) = \boxed{2270 \text{ J}}$

$q = n C_p \Delta T = (1 \text{ mol}) \left(\frac{5}{2} \cdot 0.0820 \text{ atm} \cdot \text{L} / \text{mol} \cdot \text{K} \right) (273 \text{ K}) = 56.0 \text{ atm} \cdot \text{L} \Rightarrow (56.0) (101.325 \text{ J}) = \boxed{5671 \text{ J}}$

b. Constant volume / $V_1 = V_2 = 44.8 \text{ L} \Rightarrow \Delta V = 0 \therefore W = 0$ $P_1 = 1 \text{ atm}$ $P_2 = 2 \text{ atm}$

$q_v = n C_v \Delta T = (1 \text{ mol}) \left(\frac{3}{2} \cdot 0.0820 \right) (546 \text{ K}) = \boxed{682 \text{ J}}$

$T_1 = 546 \text{ K}$

$T_2 = \frac{P_2 V_2}{nR} = \frac{(2 \text{ atm}) (44.8 \text{ L})}{(1 \text{ mol}) (0.0820)} = 1093 \text{ K}$ $\Delta T = 546 \text{ K}$

c. $w_3 = \int P dV = \int_{V_1}^{V_2} (6.643 \times 10^{-4} V^2 + 0.6667) dV$

$$= \frac{1}{3} (6.643 \times 10^{-4}) (V_2^3 - V_1^3) + 0.6667 (V_2 - V_1)$$

$$= \frac{1}{3} (6.643 \times 10^{-4}) (22.4^3 - 44.8^3) + (0.6667) (-22.4)$$

$$\approx \boxed{-3278 \text{ J}}$$

$\Delta U = 0 \therefore q_{\text{net}} = w_{\text{net}} \Rightarrow q_{\text{net}} = w_{\text{net}}$

$\Delta U_1 + \Delta U_2 + \Delta U_3 = \Delta U_{\text{net}} = 0$ $\Delta U_3 = -(\Delta U_1 + \Delta U_2) = -(3401 + 682) = -10222$

$(q_1 - w_1 = 3401)$ $(q_2 - w_2 = 682)$

$q_3 = \Delta U_3 + w_3 = -10222 + (-3278) = \boxed{-13500 \text{ J}}$

3.

3.1 The initial state of 1 mole of a monatomic ideal gas is $P = 10 \text{ atm}$ and $T = 300 \text{ K}$. Calculate the change in the entropy of the gas for

- An isothermal decrease in the pressure to 5 atm
- A reversible adiabatic expansion to a pressure of 5 atm
- A constant-volume decrease in the pressure to 5 atm

$$a. \quad V_1 = \frac{nRT_1}{P_1} = \frac{(1 \text{ mol})(0.08206)(300 \text{ K})}{10 \text{ atm}} = 2.46 \text{ L} \quad / \quad V_2 = \frac{nRT_2}{P_2} = \frac{(1 \text{ mol})(0.08206)(300 \text{ K})}{5 \text{ atm}} = 4.92 \text{ L}$$

$$\text{isothermal} \Rightarrow \Delta U = 0 \quad \therefore q = w = nRT \ln\left(\frac{V_2}{V_1}\right)$$

$$\Delta S = \frac{q}{T} = \frac{nRT \ln\left(\frac{V_2}{V_1}\right)}{T} = nR \ln\left(\frac{V_2}{V_1}\right) = (1 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K}) \ln\left(\frac{4.92}{2.46}\right) = \boxed{5.76 \text{ J/K}}$$

$$b. \quad \text{adiabatic: } q = 0$$

$$\therefore \Delta S = \int \frac{dq_{rev}}{T} = 0 \quad (\because dq = 0)$$

$$c. \quad V_1 = V_2 = 2.46 \text{ L}, \quad \Delta V = 0 \Rightarrow \therefore w = 0 \quad \left(\frac{5}{3} \times = \frac{1}{2} \times\right). \quad \therefore \Delta U = q - w = q$$

$$q_v = nC_v \Delta T \quad \left(\int q_v = nC_v dT\right) \quad / \quad T_2 = \frac{P_2 V_2}{nR} = \frac{(5 \text{ atm})(2.46 \text{ L})}{(1 \text{ mol})(0.08206)} = 150 \text{ K}$$

$$\Delta S = \int_{T_1}^{T_2} \frac{dq_v}{T} = \int_{T_1}^{T_2} \frac{1}{T} \cdot nC_v dT = nC_v \ln\left(\frac{T_2}{T_1}\right) \Rightarrow (1 \text{ mol}) \cdot \frac{5}{2} R \cdot \ln\left(\frac{150 \text{ K}}{300 \text{ K}}\right) = (1 \text{ mol}) \cdot \frac{5}{2} \cdot (8.314 \text{ J/mol}\cdot\text{K}) \cdot \ln\left(\frac{1}{2}\right) = \boxed{-8.64 \text{ J/K}}$$

4.

3.4 Calculate the change in the ΔH enthalpy and the change in ΔS entropy when 1 mole of SiC is heated from 25°C to 1000°C . The constant-pressure molar heat capacity of SiC varies with temperature as

$$[c_p = 50.79 + 1.97 \times 10^{-3}T - 4.92 \times 10^{-6}T^{-2} + 8.20 \times 10^{-8}T^{-3} \text{ J/mole} \cdot \text{K}]$$

$$T_1 = 25^\circ\text{C} \quad T_2 = 1000^\circ\text{C} \quad P_1 = P_2, \quad \Delta T = 975\text{K}$$

$$\underline{\underline{= 298\text{K}}} \quad \underline{\underline{= 1273\text{K}}}$$

i) $\Delta H = n C_p \Delta T$ ($\therefore dH = n C_p dT$) /

$$\therefore dH = (1\text{mol}) \cdot (50.79 + (1.97 \times 10^{-3}T) - (4.92 \times 10^{-6}T^{-2}) + (8.20 \times 10^{-8}T^{-3} \text{ J/mole} \cdot \text{K})) \cdot dT$$

$$\int dH = \int_{T_1}^{T_2} (50.79 + (1.97 \times 10^{-3}T) - (4.92 \times 10^{-6}T^{-2}) + (8.20 \times 10^{-8}T^{-3} \text{ J/mole} \cdot \text{K})) dT, \quad T_1 = 298\text{K} \quad T_2 = 1273\text{K}$$

$$\approx \boxed{42750 \text{ J (approx)} : \Delta H}$$

ii) $\Delta S = \int_{T_1}^{T_2} \frac{1}{T} dH$, $dH = dH$ ($\therefore$ constant pressure process)

$$\therefore \Delta S = \int_{T_1}^{T_2} \frac{1}{T} dH = \int_{T_1}^{T_2} (50.79T^{-1} + (1.97 \cdot 10^{-3}) - (4.92 \times 10^{-6}T^{-3}) + (8.20 \times 10^{-8}T^{-4} \text{ J/mole} \cdot \text{K})) dT / T_1 = 298\text{K}, T_2 = 1273\text{K}$$

$$\approx \boxed{59.7 \text{ J/K (approx)} : \Delta S}$$