

- 2.1 An monatomic ideal gas at 300 K has a volume of 15 liters at a pressure of 15 atm. Calculate

- The final volume of the system
- The work done by the system
- The heat entering or leaving the system
- The change in the internal energy
- The change in the enthalpy when the gas undergoes
 - A reversible isothermal expansion to a pressure of 10 atm
 - A reversible adiabatic expansion to a pressure of 10 atm

The constant-volume molar heat capacity of the gas, c_v , has the value $1.5 R$.

i) reversible isothermal expansion. If $P_f = 10 \text{ atm}$.

$$T_i = 300 \text{ K}, V_i = 15 \text{ L}, P_i = 15 \text{ atm} \quad / \quad \Delta T = 0 \text{ (isothermal)}$$

$$a) P_i V_i = P_f V_f \quad (\because \Delta T = 0), \quad 15 \text{ atm} \cdot 15 \text{ L} = 10 \text{ atm} \cdot V_f, \quad V_f = \frac{45}{2} \text{ L}$$

b)

$$W = \int_{V_i}^{V_f} P dV = \int_{V_i}^{V_f} \frac{nRT}{V} dV = nRT \ln\left(\frac{V_f}{V_i}\right), \quad P_i V_i = nRT = 225 \text{ atm} \cdot \text{L}$$

$$\therefore W = 225 \text{ atm} \cdot \text{L} \ln\left(\frac{45}{30}\right) = 91.2 \text{ atm} \cdot \text{L} = 9243 \text{ J}$$

$$c) U = q - w = 0, \quad U = nC_V \Delta T = 0 \quad (\because \Delta T = 0)$$

$$\therefore q = w, \quad q = 9243 \text{ J}$$

$$d) U = nC_V \Delta T = 0$$

$$e) \Delta H = nC_p \Delta T = 0 \quad (\because \Delta T = 0)$$

ii) reversible adiabatic expansion

$$P_i = 15 \text{ atm}, \quad V_i = 15 \text{ L}, \quad T_i = 300 \text{ K}, \quad / \quad P_f = 10 \text{ atm}, \quad q = 0 \text{ (adiabatic)}$$

$$a) P_i V_i^\gamma = P_f V_f^\gamma, \quad C_p = C_v + R = \frac{5}{2}R + R = \frac{7}{2}R.$$

$$\gamma = \frac{C_p}{C_v} = \frac{5}{3} \quad / \quad (15 \text{ atm})(15 \text{ L})^{\frac{5}{3}} = (10 \text{ atm})(V_f)^{\frac{5}{3}} \quad \therefore V_f = 19.1 \text{ L}$$

$$b) n = \frac{P_i V_i}{R \cdot T_i} = \frac{15 \text{ atm} \cdot 15 \text{ L}}{0.082 \text{ atm} \cdot \text{L} \cdot \text{mol}^{-1} \cdot \text{K}^{-1} \cdot 300 \text{ K}} = 9.14 \text{ mol.}$$

$$T_f = \frac{P_f V_f}{n \cdot R} = \frac{10 \text{ atm} \cdot 19.1 \text{ L}}{9.14 \text{ mol} \cdot 0.082 \text{ atm} \cdot \text{L} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}} = 255 \text{ K}$$

$$\Delta U = q - w = -w \quad (\because q = 0), \quad \therefore w = -\Delta U = -nC_V \Delta T = -9.14 \text{ mol} \times \frac{3}{2}R(255 \text{ K} - 300 \text{ K}) \\ = 5121 \text{ J}$$

$$c) q = 0 \text{ (adiabatic)}, \quad d) \Delta U = -w = -5121 \text{ J}, \quad e) \Delta H = nC_p \Delta T = 9.14 \times \frac{5}{2}R(-45 \text{ K}) \\ = -8544 \text{ J}$$

2.2 One mole of a monatomic ideal gas, in the initial state $T = 273 \text{ K}$, $P = 1 \text{ atm}$, is subjected to the following three processes, each of which is conducted reversibly:

- A doubling of its volume at constant pressure,
 - Then a doubling of its pressure at constant volume,
 - Then a return to the initial state along the path $P = 6.643 \times 10^{-4}V^2 + 0.6667$.
- Calculate the heat and work effects which occur during each of the three processes.

$$n = 1 \text{ mol}, \quad T_i = 273 \text{ K}, \quad P_i = 1 \text{ atm}$$

$$P_i V_i = nRT, \quad V_i = 22.4 \text{ L}$$

$$\text{a) Work done by gas: } \int_{V_i}^{V_f} P dV = P \int_{V_i}^{V_f} dV = 1 \text{ atm} \cdot (2V_f - V_i) = 22.4 \text{ atm} \cdot 22.4 \text{ L} = 2270 \text{ J}$$

$$T_f = \frac{2P_f V_f}{nR} = 2T_i = 546 \text{ K}, \quad \Delta U = nC_V \Delta T = 1 \text{ mol} \cdot \frac{3}{2}R \cdot (273 \text{ K}) = 3403 \text{ J}$$

$$\Delta U = q - w, \quad 3403 \text{ J} = q - 2270 \text{ J} \quad \therefore q = 5673 \text{ J}$$

$$\text{b) } V_2 = 44.8 \text{ L}, \quad T_2 = 546 \text{ K}, \quad P_i = 1 \text{ atm} \rightarrow P_f = 2 \text{ atm}$$

$$T_3 = \frac{V_2 P_f}{nR} = \frac{2V_2 P_i}{nR} = 2T_2 = 1092 \text{ K}$$

$$\Delta U = 0, \quad \therefore w = 0 \quad / \quad \Delta U = q - w = q, \quad \Delta U = nC_V \Delta T = 1 \text{ mol} \cdot \frac{3}{2}R \cdot (546 \text{ K}) = 6805 \text{ J}$$

$$\therefore q = 6805 \text{ J}$$

$$\text{c) } T_{cb} = 1092 \text{ K}, \quad P_b = 2 \text{ atm}, \quad V_b = 44.8 \text{ L} \rightarrow T_{(a)} = 273 \text{ K}, \quad P_{(a)} = 1 \text{ atm}, \quad V_{(a)} = 22.4 \text{ L}$$

Work done by gas

$$w = \int_{V_i}^{V_f} P dV = \int_{V_i}^{V_f} 6.643 \times 10^{-4} \cdot V^2 + 0.6667 \cdot dV = \left[\frac{6.643 \times 10^{-4}}{3} V^3 + 0.6667 V \right]_{44.8 \text{ L}}^{22.4 \text{ L}} = -32.35 \text{ atm} \cdot \text{L} = -3218 \text{ J}$$

$$\Delta U = nC_V \Delta T = 1 \text{ mol} \times \frac{3}{2}R \times (273 \text{ K} - 1092 \text{ K}) = q - w$$

$$\therefore q = -3486 \text{ J}$$

3.1 The initial state of 1 mole of a monatomic ideal gas is $P = 10 \text{ atm}$ and $T = 300 \text{ K}$.

Calculate the change in the entropy of the gas for

- An isothermal decrease in the pressure to 5 atm
- A reversible adiabatic expansion to a pressure of 5 atm
- A constant-volume decrease in the pressure to 5 atm

$$P_i = 10 \text{ atm}, \quad T_i = 300 \text{ K}, \quad n = 1 \text{ mol} \quad / \quad V_i = \frac{nRT}{P_i} = 2.46 \text{ L}$$

$$\text{a) } \Delta T = 0, \quad P_f = 5 \text{ atm} \quad / \quad V_f = 4.92 \text{ L}$$

work done by gas:

$$w = \int_{V_i}^{V_f} P dV = nRT \ln\left(\frac{V_f}{V_i}\right) = 1 \text{ mol} \cdot 8.31 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1} \cdot 300 \text{ K} \ln(2) = 1728 \text{ J}$$

$$\Delta U = nC_V \Delta T = 0 = q - w, \quad \therefore q = 1728 \text{ J}$$

$$\Delta S = \frac{q}{T} = \frac{1728 \text{ J}}{300 \text{ K}} = 5.76 \text{ J} \cdot \text{K}^{-1}$$

$$\text{b) adiabatic, } q = 0 \quad \therefore \Delta S = \frac{q}{T} = 0$$

$$\text{c) } \Delta V = 0, \quad P_f = 5 \text{ atm} \quad / \quad T_f = \frac{1}{2} T_i = 150 \text{ K}, \quad w = 0 \quad \therefore \Delta U = nC_V \Delta T = q$$

$$\Delta S = \int \frac{dq}{T} = \int_{T_i}^{T_f} \frac{nC_V}{T} dT = nC_V \ln\left(\frac{T_f}{T_i}\right) = 1 \text{ mol} \times \frac{3}{2} R \times \ln\left(\frac{1}{2}\right) = -8.65 \text{ J} \cdot \text{K}^{-1}$$

3.4 Calculate the change in the enthalpy and the change in entropy when 1 mole of SiC is heated from 25°C to 1000°C. The constant-pressure molar heat capacity of SiC varies with temperature as

$$c_p = 50.79 + 1.97 \times 10^{-3}T - 4.92 \times 10^6 T^{-2} + 8.20 \times 10^8 T^{-3} \text{ J/mole} \cdot \text{K}$$

$$\Delta H = \int_{T_i}^{T_f} nC_p dT = \int_{298 \text{ K}}^{1273 \text{ K}} 1 \text{ mol} \cdot c_p dT$$

$$= \left[50.79T + \frac{1.97 \times 10^{-3}}{2} T^2 + 4.92 \times 10^6 T^{-1} - \frac{8.20 \times 10^8}{2} T^{-2} \right]_{298}^{1273}$$

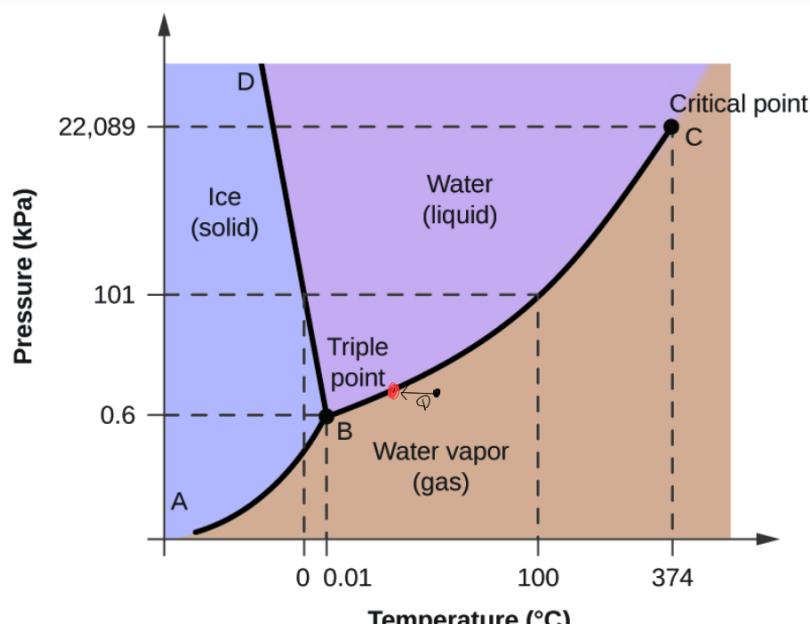
$$= 49520 + 1509 + (-12645) + (4364)$$

$$= \underline{\underline{42150 \text{ J}}}$$

$$\Delta S = \int_{T_i}^{T_f} \frac{dq}{T} = \int_{T_i}^{T_f} \frac{nC_p}{T} dT$$

$$= \left[50.79 \ln(T) + \frac{1.97 \times 10^{-3}}{2} T + \frac{4.92 \times 10^6}{2} \frac{1}{T^2} - \frac{8.20 \times 10^8}{3} \frac{1}{T^3} \right]_{298 \text{ K}}^{1273 \text{ K}}$$

$$= \underline{\underline{59.7 \text{ J} \cdot \text{K}^{-1}}}$$



김서림은 $gas \rightarrow liquid$ 의 과정이다.
늦가을은 차内外의 온도가 떠
놓고, 차内外의 습증기가 차내에
창문 쪽에서 ①의 과정을 거쳐
김이 서리게 된다.

만약 이를 해결하기 위해 난방장치를 이용하면 더 높은 습증기가 시간이 지나 창이 다시 차가워지면 김이 서리게 되며 더 높은 김이 서린다. 따라서 에어컨을 이용하여 창밖과 내부의 온도 차이를 줄이는 것이 더 현명하다. (추가로, 에어컨은 제습 기능도 있다.)

6. 가치가 퍼져나가는 것은 gas 가 운동할 확률이 가장 높은 배치가 되는 것이다. 가치 입자 개수의 scale이 매우 커 균일하게 퍼질 가능성이 양도적으로 높다. 따라서 가치 입자를 온 배치 확률이 가장 높은, 안정한 배치를 위해 자발적으로 이용하게 될 것이고 자발적으로 움직이는 것은 퍼져 나가는 driving force가 있는 것처럼 보이게 된다.

7. (6)의 예시를 들면, 입자 하나하나는 무작위적인 움직임을 하고 입자 각각은 계속 원쪽에 모여할 수 있으나 전체적으로 gas 가 퍼져나가고 이는 비가역적이다. 이처럼 입자 하나하나는 확률적으로 동일한 reversible 인데, 개수의 scale이 매우 커서 가치가 가장 확률이 높은 경우 이외의 경우가 나타날 확률이 거의 0이 가까워 거시적으로 irreversible 한 것을 말한다.

즉, 개수 scale이 매우 커서 확률 그래프가 매우 sharp 한 것을 말한다.