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a)

$$G_m = X_A G_A^\circ + X_B G_B^\circ + RT(X_A \ln X_A + X_B \ln X_B) + \Omega X_A X_B$$

Since $f = \frac{G_m}{V_m}$ then $f''(X_B) = \frac{G_m''}{V_m}$

thus $G_m' = +G_B - G_A + RT[-\ln(1-X_B) + \ln X_B + 1 + (1-X_B)(-\frac{1}{1-X_B}) +$
$+ (1-2X_B)\Omega$

then $G_m'' = RT\left(\frac{1}{1-X_B} + \frac{1}{X_B}\right) - 2\Omega$ & $1-X_B = X_A$

$f''(X_B) + \frac{2E\eta^2}{1-\nu} = 0$ thus $\frac{\left(RT\left(\frac{1}{X_A} + \frac{1}{X_B}\right) - 2\Omega\right)}{V_m} + \frac{2E\eta^2}{1-\nu} = 0$

$$T_c = \left(\frac{-2E\eta^2 \cdot V_m}{1-\nu} + 2\Omega\right) \Big/ R\left(\frac{1}{X_A} + \frac{1}{X_B}\right)$$

$V_m = V_A X_A + V_B X_B = \frac{M_A}{\rho_A} X_A + \frac{M_B}{\rho_B} X_B = 9X_A + 10X_B$

→ by plugging values in equation

for $\eta = 0$ → term $\frac{E\eta^2}{1-\nu} \cdot V_m = 0$ → thus only 2Ω remins

$$\rightarrow T_C = \frac{3 \times 10^4}{R\left(\frac{1}{X_A} + \frac{1}{X_B}\right)}$$

for $\eta = 0.06 \rightarrow \dfrac{3\times10^4 - \frac{2\times10^{11}}{1-0.3}\cdot(0.06)^2\cdot(9X_A + 10X_B)}{R\left(\frac{1}{X_A}+\frac{1}{X_B}\right)} = T_C$

b) plugging in values of X_A, X_B

$\eta = 0 \rightarrow T_C = \dfrac{3\times10^4}{RC\left(\frac{1}{X_A}+\frac{1}{X_B}\right)} \rightarrow$ for $X_B = 0.75 \rightarrow T_C \simeq 677k$

for $X_B = 0.6 \rightarrow T_C = 866k$

$\eta = 0.06 \rightarrow$ from ★ $\rightarrow$ for $X_B = 0.75 \rightarrow T_C = 450k$

for $X_B = 0.6 \rightarrow T_C = 580.2$

c) $B^2 \leq -\frac{1}{2k}\left[f''(X_B) + \dfrac{2E\eta^2}{1-\nu}\right]$ I

$\rightarrow B_C^2 = -\frac{1}{2k}\left[f''(X_B) + \dfrac{2E\eta^2}{1-\nu}\right] \rightarrow$ by plugging values in and using equation obtained fo f'' we have

$X_B = 0.75 : \eta = 0 \rightarrow B_C^2 = -2.2\times10^{17} \rightarrow$ negative

$\eta = 0.06 \rightarrow B_C^2 = -7.38\times10^{17} < 0$

$\hookrightarrow$ No sponidal decomposition

$X_B = 0.6$ for $\eta = 0 \rightarrow \beta_c^2 = 1.6 \times 10^{+17} \rightarrow \beta_c = 4 \times 10^{8}$

Since $\beta = \frac{2\pi}{\lambda} \rightarrow \lambda = 1.57 \times 10^{-8}\, m$

for $\eta = 0.06 \rightarrow \beta_c^2 = -3.5 \times 10^{17} < 0 \rightarrow$ NO spinodal

d) it happens at $X_B = X_A = 0.5$ (maximum instability)

by plugging in values at I

for $\eta = 0 \rightarrow \beta_c^2 = 22 \times 10^{16} \rightarrow \beta_c = 4.7 \times 10^{8}$

$\rightarrow \lambda_c = 1.31 \times 10^{-8}\, m \rightarrow \lambda_{max} = \sqrt{2}\lambda_c = 1.85 \times 10^{-8}\, m$

for $\eta = 0.06 \rightarrow \beta_c^2 = -2.93 \times 10^{17} < 0 \rightarrow$ no spinodal

e) $R(\beta) = -M\beta^2 \left[f''(X_B) + \frac{2E\eta^2}{1-\nu} + 2k\beta^2 \right] > 0$

Since $M = X_A X_B (X_A \cdot D_B^* + X_B D_A^*)$

$\downarrow$

maximum (at $X_A = X_B = 0.5$) thus $M_{max} = 4.5 \times 10^{-11}$

thus at $\eta = 0 \rightarrow R(\beta) = 1.13 \times 10^{15} \rightarrow$ maximum anywhere but at $\eta = 0.06$ spinodal will not occur

2)

a) $C_A(x,10) = C_A(x,0)\exp(-\pi^2 \cdot D \times 10/\lambda^2) = 0.88\, C_A(x,0)$

&

$C_A(x,100) = C_A(x,0)\exp(-\pi^2 \cdot D \times 100/\lambda^2) = 0.29\, C_A(x,0)$

$\rightarrow \dfrac{C_A(x,10)}{C_A(x,100)} = 0.3$ or $\dfrac{C_A(x,100)}{C_A(x,10)} = 3 \rightarrow$ it increases

b) $C(x,100)\Big)_{\lambda = 0.1\,\mu m} = 0.99\, C_A(x,0)$ it decrease

$\uparrow$

$C(x,100)\Big)_{\lambda = 0.01\,\mu m} = 0.29 \longrightarrow \dfrac{C(\text{at } 0.01)}{C(\text{at } 0.1)} = 0.292$

c) $D_A(T=298) = 10^{-4}\exp\left(-\dfrac{85000}{8.314 \times 298}\right) = 1.26 \times 10^{-19}$

$D_A(T=398) = 7 \times 10^{-16}$

$$\frac{C_{100}(\text{at } T=398)}{C_{100}(\text{at } T=298)} = \frac{C_A(x,0)}{C_A(x,0)} \times \frac{\exp(-\pi^2 \cdot 7 \times 10^{-16}/\lambda^2)}{\exp(-\pi^2 \times 1.26 \times 10^{-19}/\lambda^2)} \simeq 0$$

→ extreme decrease

d) Tempreture has highest effect
(T)