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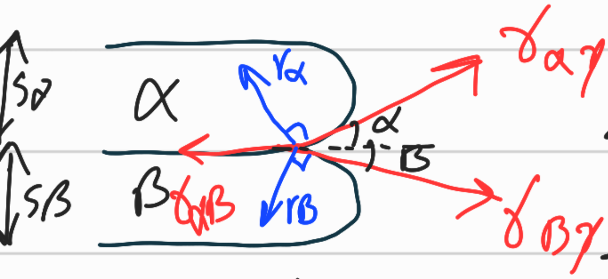
Q1:

a) energy balance at interface is important



I $\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\alpha + \gamma_{\beta\gamma}\cos\beta$

II $\gamma_{\alpha\gamma}\sin\alpha = \gamma_{\beta\gamma}\sin\beta$

only if $\gamma_{\alpha\beta} = 0$ then $\alpha = \beta = 90°$ but since $\gamma_{\alpha\beta} \neq 0 \rightarrow \alpha, \beta \neq 90$

$\longrightarrow$ curvature exists for energy balance

b) let's call α, β angles $\theta_\alpha, \theta_\beta$ to avoide confusion

we will have

$$S_\alpha = 2r_\alpha \cdot \sin\left(\frac{\pi}{2} - \theta_\alpha\right) = 2r_\alpha\cos\theta_\alpha$$

similarly $S_\beta = 2r_\beta \cdot \cos\theta_\beta$

now: $\gamma_{\alpha\gamma}\sin\theta_\alpha = \gamma_{\alpha\beta}\sin\theta_\beta \longrightarrow \gamma_{\alpha\gamma}^2 \cdot \sin^2\theta_\alpha = \gamma_{\alpha\beta}^2 \cdot \sin^2\theta_\beta$

$$\longrightarrow \gamma_{\alpha\gamma}^2(1 - \cos\theta_\alpha) = \gamma_{\beta\gamma}^2(1 - \cos^2\theta_\beta) \Longrightarrow$$

$$\cos\theta_\beta = \sqrt{1 - \left(\frac{\gamma_{\alpha\gamma}}{\gamma_{\beta\gamma}}\right)^2(1 - \cos^2\theta_\alpha)}$$

$$\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\theta_\alpha + \gamma_{\beta\gamma}\cos\theta_\beta = \gamma_{\alpha\gamma}\cos\theta_\alpha + \gamma_{\beta\gamma}\sqrt{1 - \left(\frac{\gamma_{\alpha\gamma}}{\gamma_{\beta\gamma}}\right)^2(1 - \cos^2\theta_\alpha)}$$

$$\gamma_{\alpha\beta} - \gamma_{\alpha\gamma}\cos\theta_\alpha = \gamma_{\beta\gamma}\sqrt{1 - \left(\frac{\gamma_{\alpha\gamma}}{\gamma_{\beta\gamma}}\right)^2(1 - \cos^2\theta_\alpha)}$$

$\gamma_{\alpha\beta}^2 - 2\gamma_{\alpha\beta}\gamma_{\alpha\gamma}\cos\theta_\alpha + \gamma_{\alpha\gamma}^2\cos^2\theta_\alpha = \gamma_{\beta\gamma}^2 -$
$\gamma_{\alpha\gamma}^2(1-\cos^2\theta_\alpha)$

$$\Longrightarrow \cos\theta_\alpha \overset{?}{=} \frac{\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2}{2\gamma_{\alpha\beta}\gamma_{\alpha\gamma}}$$

thus: $r_\alpha = \dfrac{S_\alpha}{2\cos\theta_\alpha} = \dfrac{S_\alpha\gamma_{\alpha\beta}\gamma_{\alpha\gamma}}{\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2}$ ✓ (III)

similarly

$$r_\beta = \frac{S_\beta\gamma_{\alpha\beta}\gamma_{\beta\gamma}}{\gamma_{\alpha\beta}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2} \checkmark \text{ (IV)}$$

c) $\Delta G = \dfrac{\gamma_{\alpha\gamma}}{r_\alpha}V_\alpha + \dfrac{\gamma_{\beta\gamma}}{r_\beta}V_\beta \longrightarrow$ by replacing r_α & r_β

and the fact that $\dfrac{r_\alpha}{S_\alpha} = \dfrac{V_\beta}{S_\beta} = \dfrac{V_m}{S}$ we will have:

$$\Delta G = \left[\frac{\cancel{S_\alpha}\cancel{\gamma_{\alpha\gamma}}\cancel{(\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2)}}{\cancel{S_\alpha}\gamma_{\alpha\beta}\cancel{\gamma_{\alpha\gamma}}} + \frac{\cancel{S_\beta}\cancel{\gamma_{\beta\gamma}}\cancel{(\gamma_{\alpha\beta}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2)}}{\cancel{S_\beta}\gamma_{\alpha\beta}\cancel{\gamma_{\beta\gamma}}}\right]^2 \times \frac{V_m}{S}$$

$$\longrightarrow \Delta G = \frac{2\gamma_{\alpha\beta}V_m}{S}$$

Q2:

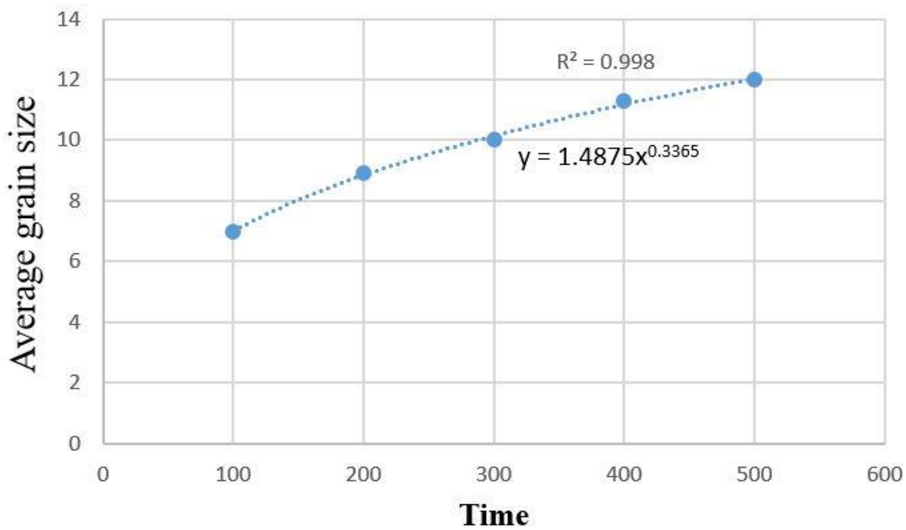
a)

For 64000 grains

100 each step

And T= 100 we will have

Since $R=kt^n$ then n=0.3365



b)

$$R = kt^n = k_0 \left\{ e^{-\frac{Q}{kT}} \right\} t^n$$

$$\ln R = -\frac{Q}{k} \times \frac{1}{T} + \ln(\underbrace{K_0 t^n}_{C})$$

$$\downarrow$$

$$\underbrace{-\frac{Q}{k}}_{slope} \cdot \frac{1}{T} + \ln C \rightarrow$$ by this graph (5-10-15-20 for T) 64000 grains

we have $-\frac{Q}{k} = -1.41 \rightarrow Q \simeq 11.7$