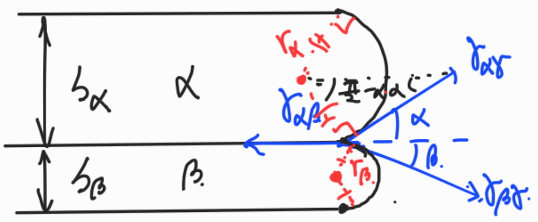
HW6 20222571 최찬식

1. a). The interfaces have interface energy and the energies are balanced state.



Young's equation.

① $\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\alpha + \gamma_{\beta\gamma}\cos\beta$.

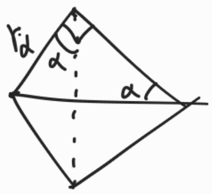
② $\dfrac{\gamma_{\alpha\gamma}}{\sin(\pi-\beta)} = \dfrac{\gamma_{\beta\gamma}}{\sin(\pi-\alpha)} \rightarrow \gamma_{\alpha\gamma}\sin\alpha = \gamma_{\beta\gamma}\sin\beta$.

the lamella structure is maintained with curved tip.

$$\gamma_{\alpha\gamma}\sin\alpha = \gamma_{\beta\gamma}\sin\beta.$$

$\gamma_{\alpha\gamma}, \gamma_{\beta\gamma}, \gamma_{\alpha\beta} \neq 0$ therefore $\alpha, \beta \neq 0$.

b). S_α, S_β. can be expressed as following equation.



$$S_\alpha = 2r_\alpha\cos\alpha, \quad S_\beta = 2r_\beta\cos\beta.$$

$$(\gamma_{\alpha\beta} - \gamma_{\beta\gamma}\cos\beta)^2 = (\gamma_{\alpha\gamma}\cos\alpha)^2$$

$$\gamma_{\alpha\beta}^2 + \gamma_{\beta\gamma}^2(1-\cancel{\sin^2\beta}) - 2\gamma_{\alpha\beta}\gamma_{\beta\gamma}\cos\beta = \gamma_{\alpha\gamma}^2(1-\cancel{\sin^2\alpha})$$

$$\gamma_{\alpha\beta}^2 + \gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2 = 2\gamma_{\alpha\beta}\gamma_{\beta\gamma}\cos\beta.$$

$$\therefore\ r_\beta = \frac{S_\beta}{2\cos\beta} = \frac{S_\beta\cdot\gamma_{\alpha\beta}\gamma_{\beta\gamma}}{\gamma_{\alpha\beta}^2 + \gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2}$$

$$r_\alpha = \frac{S_\alpha\cdot\gamma_{\alpha\beta}\gamma_{\alpha\gamma}}{\gamma_{\alpha\beta}^2 + \gamma_{\alpha\gamma}^2 - \gamma_{\beta\gamma}^2}$$

c). Gibbs energy change induced by capillary effect : ΔG.

$$\Delta G = \frac{\gamma_{\alpha\gamma}}{r_\alpha}V_\alpha + \frac{\gamma_{\beta\gamma}}{r_\beta}V_\beta.$$

$$= \frac{\cancel{\gamma_{\alpha\gamma}}(\gamma_{\alpha\beta}^2 + \gamma_{\alpha\gamma}^2 - \gamma_{\beta\gamma}^2)}{S_\alpha\gamma_{\alpha\beta}\cancel{\gamma_{\alpha\gamma}}}V_\alpha + \frac{\cancel{\gamma_{\beta\gamma}}(\gamma_{\alpha\beta}^2 + \gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2)}{S_\beta\gamma_{\alpha\beta}\cancel{\gamma_{\beta\gamma}}}V_\beta.$$

as volume is proportional to thickness $V = b \cdot h \cdot l$.

for both α and β. $\frac{V}{b} = h \cdot l$. $\frac{V_\alpha}{b_\alpha} = \frac{V_\beta}{b_\beta} = \frac{V_m}{b}$.

$$\Delta G = \frac{V_m}{b}\left(\frac{\gamma_{\alpha\beta}^2}{\gamma_{\alpha\beta}}\right)$$

$$\therefore \quad \Delta G = \frac{2\gamma_{\alpha\beta} V_m}{b}.$$

2. Plot setup shown below.

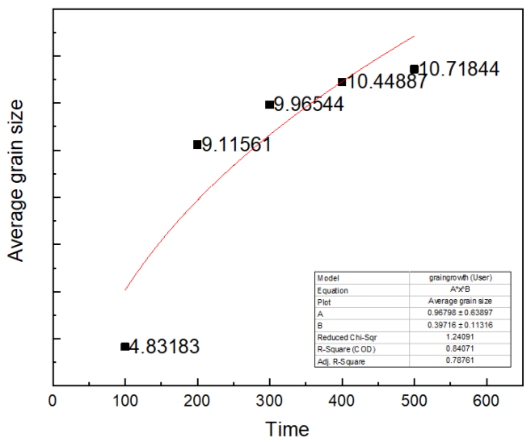
time. 500. step 100.

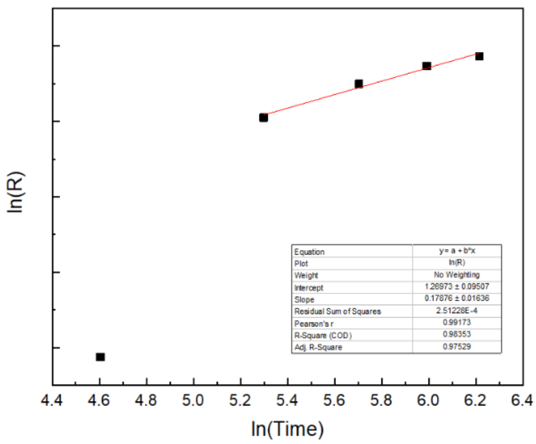
Temperature 5. 10. 15. 20.

a) General kinetics estimates average grain size $\bar{R}$ with relationship between time as.

$T=1$. $\bar{R} = k t^n$, $\ln \bar{R} = n \ln t + \ln k$.

It shows linear dependence when plotted $\ln \bar{R}$ vs. $\ln t$, the slope is n.





by omitting initial time data, the graph shows linear relationship. $n \sim 0.18$.

With time 100. the grains do not sufficiently react with adjacent ones.

b). General kinetics.

$\bar{R} = k t^n$ as proved by above answer.

$$\bar{R} = k_0 \left[\exp\left(-\frac{Q}{k_B T}\right)\right] t^n.$$

$\ln \bar{R} = \ln k_0 - \frac{Q}{k_B T} + n \ln t$, plotted as $\ln \bar{R}$ vs $\frac{1}{T}$.

slope is. $-\frac{Q}{k_B} = -1.46$ K.

$\therefore Q = 12.1 \ J/mol.$

logarithm of R is inversely proportional to T.