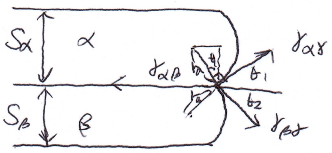
Phase transformations Problem Set #6 20222102 이다태

a)

interfacial energy 들이 force balance를 이루어야 한다.



$$\gamma_{\alpha\beta} = \cos\theta_1\,\gamma_{\alpha\gamma} + \cos\theta_2\,\gamma_{\beta\gamma}$$

$$\gamma_{\alpha\gamma}\sin\theta_1 = \gamma_{\beta\gamma}\sin\theta_2$$

위의 수식에서 curvature 가 없다고 하면 $\theta_1 = \theta_2 = 90°$ $\Rightarrow$ $\cos\theta_1 = \cos\theta_2 = 0$ 이 되며

$\gamma_{\alpha\beta} = \cos\theta_1\,\gamma_{\alpha\gamma} + \cos\theta_2\,\gamma_{\beta\gamma} = 0$ 이 된다.

하지만 α와 β 사이의 interfacial energy 가 0이 되면 안되므로 curvature 가 있어야 한다.

b) i) $S_\alpha = 2r_\alpha\cos\theta_1$ $\qquad S_\beta = 2r_\beta\cos\theta_2$

$$\gamma_{\alpha\gamma}\sin\theta_1 = \gamma_{\beta\gamma}\sin\theta_2$$

$$\Rightarrow \gamma_{\alpha\gamma}^2\sin^2\theta_1 = \gamma_{\beta\gamma}^2\sin^2\theta_2$$

$$\Rightarrow \gamma_{\alpha\gamma}^2(1-\cos^2\theta_1) = \gamma_{\beta\gamma}^2(1-\cos^2\theta_2)$$

$$\cos\theta_2 = \sqrt{\frac{\gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\alpha\gamma}^2\cos^2\theta_1}{\gamma_{\beta\gamma}^2}}$$

ii) $\gamma_{\alpha\beta} = \cos\theta_1\,\gamma_{\alpha\gamma} + \cos\theta_2\,\gamma_{\beta\gamma}$

i) 결과 대입

$$\Rightarrow \gamma_{\alpha\beta} = \cos\theta_1\,\gamma_{\alpha\gamma} + \cancel{\gamma_{\beta\gamma}}\sqrt{\frac{\gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\alpha\gamma}^2\cos^2\theta_1}{\cancel{\gamma_{\beta\gamma}^2}}}$$

$$= \frac{S_\alpha}{2r_\alpha}\gamma_{\alpha\gamma} + \sqrt{\gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\alpha\gamma}^2\frac{S_\alpha^2}{4r_\alpha^2}}$$

iii) 항 옮겨서 제곱.

$$\gamma_{\alpha\beta}^2 - \frac{S_\alpha}{r_\alpha}\gamma_{\alpha\beta}\gamma_{\alpha\gamma} + \cancel{\frac{S_\alpha^2}{4r_\alpha^2}\gamma_{\alpha\gamma}^2} = \gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2 + \cancel{\frac{S_\alpha^2}{4r_\alpha^2}\gamma_{\alpha\gamma}^2}$$

$$\Rightarrow \frac{1}{r_\alpha} = \frac{\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2}{S_\alpha\gamma_{\alpha\beta}\gamma_{\alpha\gamma}}$$

$$\therefore r_\alpha = \frac{S_\alpha\gamma_{\alpha\beta}\gamma_{\alpha\gamma}}{\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2}$$

똑같은 방식으로 r_β를 구하면

$$r_\beta = \frac{S_\beta\gamma_{\alpha\beta}\gamma_{\beta\gamma}}{\gamma_{\alpha\beta}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2}$$

c) Capillary effect 만 고려했을 때

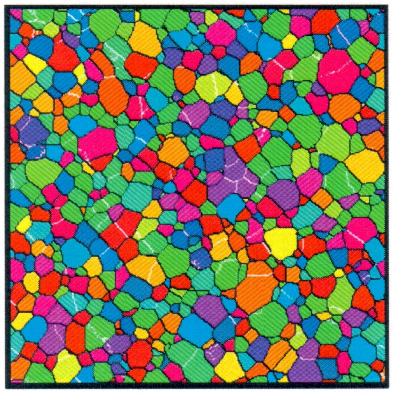
$$\Delta G = \frac{\gamma_{\alpha\gamma}}{r_\alpha} V_\alpha + \frac{\gamma_{\beta\gamma}}{r_\beta} V_\beta$$

$V_\alpha = \frac{S_\alpha}{S} V_m^\gamma$, $V_\beta = \frac{S_\beta}{S} V_m^\gamma$ 라고 가정하면

$$\Delta G = \frac{\gamma_{\alpha\gamma}\left(\gamma_{\alpha\beta}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2\right)V_\alpha}{S_\alpha \gamma_{\alpha\beta} \gamma_{\alpha\gamma}} + \frac{\gamma_{\beta\gamma}\left(\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2\right)V_\beta}{S_\beta \gamma_{\alpha\beta} \gamma_{\beta\gamma}}$$

$$= \frac{2\gamma_{\alpha\beta} V_m^\gamma}{S} = \Delta G_{IF}$$

2.



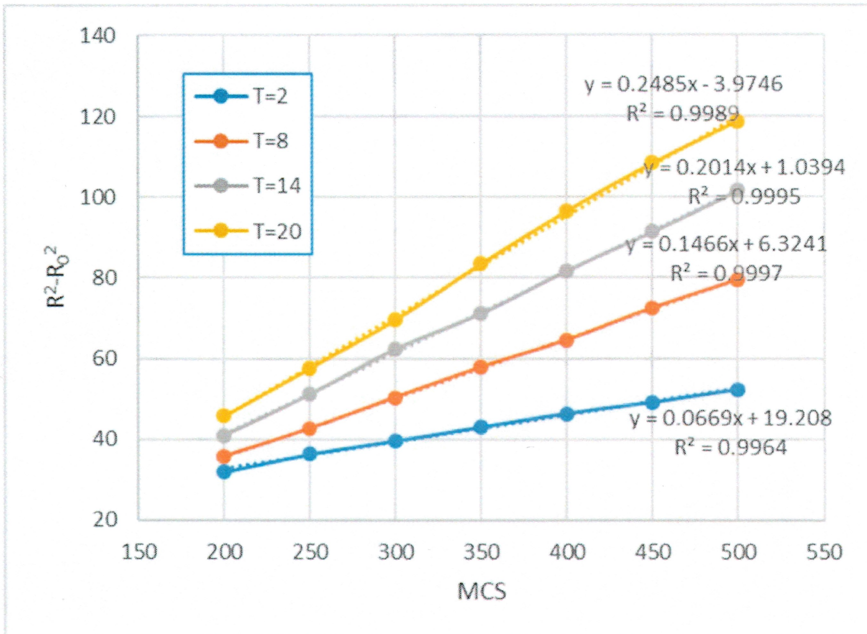
▲ 시작샘플(12100 grains)

T	k
2	0.0669
8	0.1466
14	0.2014
20	0.2485

a) $R = kt^n$ 에서 $n = 0.5$ 라 하면

$R^2 - R_0^2 = kt$ 라 할 수 있다.

온도에 따라 다음 그래프가 얻어진다.

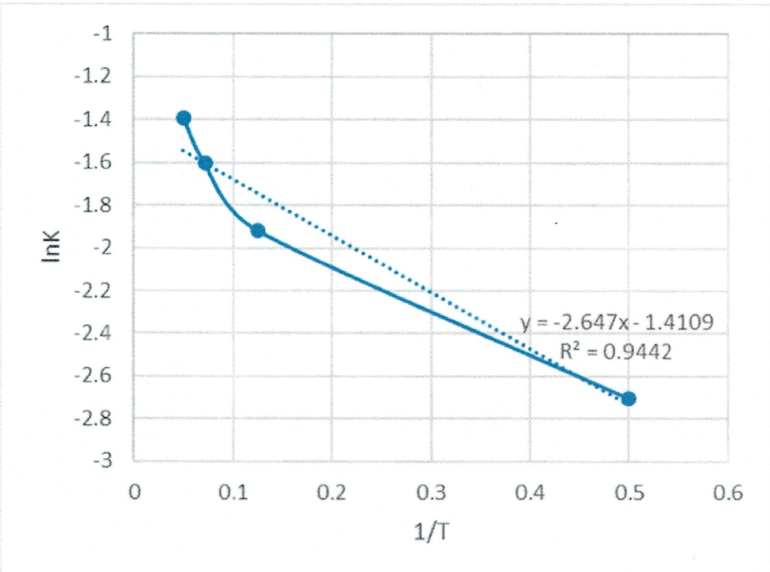


그래프가 선형을 보이므로 R과 t의 관계는 $R = kt^{\frac{1}{2}}$ 이라 할 수 있다.

b) grain size R과 온도 T는 다음 관계를 가진다.

$R^2 - R_0^2 = kt = k_0 \exp\left(-\frac{Q}{RT}\right)t$

=) 온도가 커질수록 grain growth 가 빨라진다.



$k = k_0 \exp\left(-\frac{Q}{RT}\right)$

$\ln k = -\frac{Q}{R}\frac{1}{T} + \ln k_0$ => $\frac{Q}{R} = 2.647$