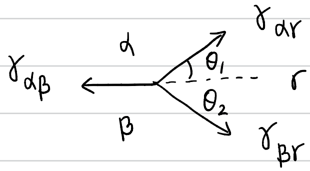
상변태 HW#6 202222341 이지수

1. (a) tip layer의 curvature



interface 에서의 force balance eqn.

$$\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\theta_1 + \gamma_{\beta\gamma}\cos\theta_2 \quad \cdots ①$$

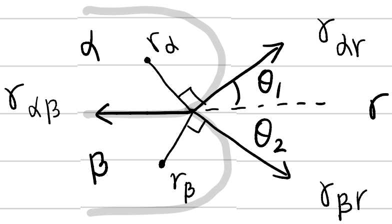
$$\gamma_{\alpha\gamma}\sin\theta_1 = \gamma_{\beta\gamma}\sin\theta_2 \quad \cdots ②$$

curvature가 없다면 $(\theta_1 = \theta_2 = 90°)$

$\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cdot 0 + \gamma_{\beta\gamma}\cdot 0 = 0 \rightarrow \gamma_{\alpha\beta} \neq 0$ 이므로 모순

따라서 layer tip은 curvature를 가져야 한다

(b) α와 β의 radius를 각각 r_α, r_β



$$S_\alpha = 2r_\alpha \sin\left(\frac{\pi}{2} - \theta_1\right) = 2r_\alpha \cos\theta_1$$

$$S_\beta = 2r_\beta \sin\left(\frac{\pi}{2} - \theta_2\right) = 2r_\beta \cos\theta_2$$

$$\Rightarrow \cos\theta_1 = \frac{S_\alpha}{2r_\alpha}, \quad \cos\theta_2 = \frac{S_\beta}{2r_\beta}$$

$$\gamma_{\alpha\gamma}\sin\theta_1 = \gamma_{\beta\gamma}\sin\theta_2 \Rightarrow \gamma^2_{\alpha\gamma}(1-\cos^2\theta_1) = \gamma^2_{\beta\gamma}(1-\cos^2\theta_2)$$

$$\Rightarrow \cos\theta_2 = \sqrt{\frac{\gamma^2_{\beta\gamma} - \gamma^2_{\alpha\gamma} + \gamma^2_{\alpha\gamma}\cos^2\theta_1}{\gamma^2_{\beta\gamma}}}$$

$$\Rightarrow \gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\theta_1 + \gamma_{\beta\gamma}\sqrt{\frac{\gamma^2_{\beta\gamma} - \gamma^2_{\alpha\gamma} + \gamma^2_{\alpha\gamma}\cos^2\theta_1}{\gamma^2_{\beta\gamma}}}, \quad \cos\theta_1 = \frac{S_\alpha}{2r_\alpha}$$

$$\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\frac{S_\alpha}{2r_\alpha} + \sqrt{\gamma^2_{\beta\gamma} - \gamma^2_{\alpha\gamma} + \gamma^2_{\alpha\gamma}\frac{S_\alpha^2}{4r_\alpha^2}}$$

$$\gamma_{\beta r}^2 - \gamma_{\alpha r}^2 + \gamma_{\alpha r}^2 \cancel{\frac{S_\alpha^2}{4r_\alpha^2}} = \gamma_{\alpha\beta}^2 + \gamma_{\alpha r}^2 \cancel{\frac{S_\alpha^2}{4r_\alpha^2}} - 2\gamma_{\alpha\beta}\gamma_{\alpha r}\frac{S_\alpha}{2r_\alpha}$$

r_α에 대해 정리하면 $r_\alpha = \dfrac{S_\alpha \gamma_{\alpha\beta}\gamma_{\alpha r}}{\gamma_{\alpha\beta}^2 - \gamma_{\beta r}^2 + \gamma_{\alpha r}^2}$

같은 방법으로 $\cos\theta_2$, S_β에 대해 계산하고 r_β에 대해 정리하면

$$r_\beta = \frac{S_\beta \gamma_{\alpha\beta}\gamma_{\beta r}}{\gamma_{\alpha\beta}^2 - \gamma_{\alpha r}^2 + \gamma_{\beta r}^2}$$

(c) $\Delta G_{capillary} = \dfrac{\gamma_{\alpha r} V_\alpha}{r_\alpha} + \dfrac{\gamma_{\beta r} V_\beta}{r_\beta}$ ← (b)의 결과 대입

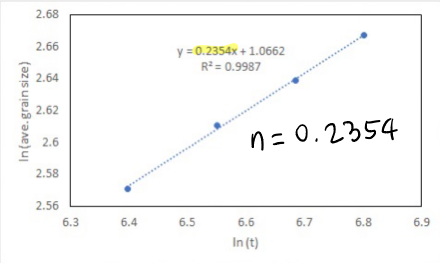
$$= \frac{\gamma_{\alpha r}(\gamma_{\alpha\beta}^2 - \gamma_{\alpha r}^2 + \gamma_{\beta r})^2 V_\alpha}{S_\alpha \gamma_{\alpha\beta}\gamma_{\alpha r}} + \frac{\gamma_{\beta r}(\gamma_{\alpha\beta}^2 - \gamma_{\beta r}^2 + \gamma_{\alpha r}^2)V_\beta}{S_\beta \gamma_{\alpha\beta}\gamma_{\beta r}}$$

$\dfrac{V_\alpha}{S_\alpha} = \dfrac{V_\beta}{S_\beta} = \dfrac{V_m}{S}$로 가정시

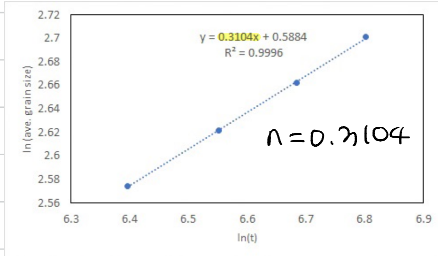
$$\Delta G_{capillary} = \frac{2\gamma_{\alpha\beta} V_m}{S} = \Delta G_{Interface}$$

2. (a) Average grain size $\bar{R} = k \cdot t^n$ (t : MCS) $\rightarrow \ln(\bar{R}) = n\ln(t) + \ln k$

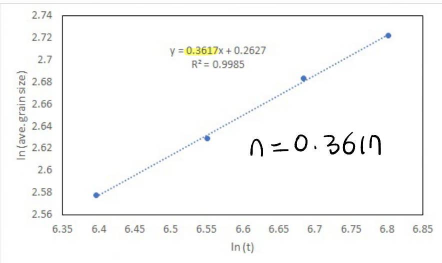
i) T=5



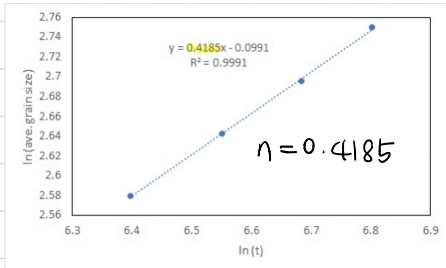
ii) T=10



iii) T=15



iv) T=20



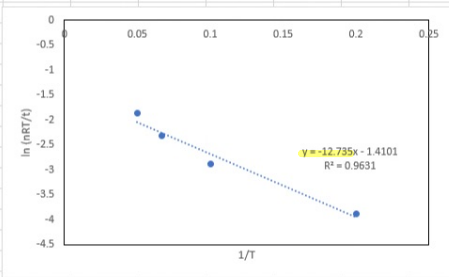
(b) $u = \lambda v = \lambda V_0 \exp(-\Delta G^*/RT)(1 - \exp(-\Delta G_{df}/RT))$, when $\Delta G_{df} \ll RT$

$$\simeq V_0 \exp(\Delta G^*/RT) \cdot \frac{\Delta G_{df}}{RT}$$

$$u = \frac{d\bar{R}}{dt} = n \cdot k \cdot t^{n-1} = \lambda V_0 \exp(\Delta G^*/RT) \cdot \frac{\Delta G_{df}}{RT}$$

$$\ln(T \cdot n \cdot k \cdot t^{n-1}) = \ln C - \frac{\Delta G^*}{RT} \rightarrow \ln\left(\frac{n\bar{R}T}{t}\right) = \ln C - \frac{\Delta G^*}{RT}$$

T	5	10	15	20		t=800
n	0.2354	0.3104	0.3617	0.4185		
R	13.99744	14.32701	14.63362	14.81829		
1/T	0.2	0.1	0.066667	0.05		
nRT/t	0.020594	0.055589	0.099243	0.155036		
ln(nRT/t)	-3.88277	-2.88977	-2.31018	-1.8641		



$$\Rightarrow \frac{\Delta G^*}{R} = 12.735$$