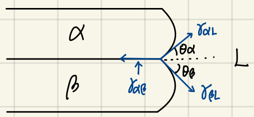
20212145 채민지 HW6

1. (a) interfacial energy 가 force balance를 이루기 때문이다.



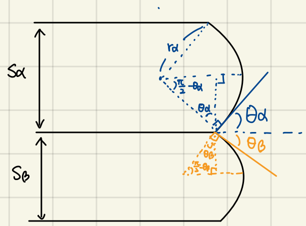
Young's eqn 에 의해,

$\gamma_{\alpha\beta} = \gamma_{\alpha L}\cos\theta_\alpha + \gamma_{\beta L}\cos\theta_\beta$,

$\gamma_{\alpha L}\sin\theta_\alpha = \gamma_{\beta L}\sin\theta_\beta$

$\gamma_{\alpha L} \neq \gamma_{\beta L} \neq \gamma_{\alpha\beta} \neq 0$ 이므로 $0 < \theta_\alpha, \theta_\beta < 90°$ 이다. 따라서 curvature를 갖는다.

(b)



$S_\alpha = 2r_\alpha\cos\theta_\alpha$, $S_\beta = 2r_\beta\cos\theta_\beta$

(a)에서 $\gamma_{\alpha L}\sin\theta_\alpha = \gamma_{\beta L}\sin\theta_\beta$

$\gamma_{\alpha L}^2\sin^2\theta_\alpha = \gamma_{\beta L}^2\sin^2\theta_\beta$

$\gamma_{\alpha L}^2(1-\cos^2\theta_\alpha) = \gamma_{\beta L}^2(1-\cos^2\theta_\beta)$

$\gamma_{\alpha\beta} = \gamma_{\alpha L}\cos\theta_\alpha + \gamma_{\beta L}\cos\theta_\beta$

$$(\gamma_{\alpha\beta} - \gamma_{\alpha L}\cos\theta_\alpha)^2 = \gamma_{\alpha\beta}^2 - 2\gamma_{\alpha\beta}\gamma_{\alpha L}\cos\theta_\alpha + \gamma_{\alpha L}^2\cos^2\theta_\alpha = \gamma_{\beta L}^2\cos^2\theta_\beta = \gamma_{\beta L}^2 - \gamma_{\alpha L}^2 + \gamma_{\alpha L}^2\cos^2\theta_\alpha$$

$$2\gamma_{\alpha\beta}\gamma_{\alpha L}\cos\theta_\alpha = \gamma_{\alpha L}^2 - \gamma_{\beta L}^2 + \gamma_{\alpha\beta}^2$$

$$\cos\theta_\alpha = \frac{\gamma_{\alpha L}^2 - \gamma_{\beta L}^2 + \gamma_{\alpha\beta}^2}{2\gamma_{\alpha\beta}\gamma_{\alpha L}}$$

$$(\gamma_{\alpha\beta} - \gamma_{\beta L}\cos\theta_\beta)^2 = \gamma_{\alpha\beta}^2 - 2\gamma_{\alpha\beta}\gamma_{\beta L}\cos\theta_\beta + \gamma_{\beta L}^2\cos\theta_\beta^2 = \gamma_{\alpha L}^2\cos^2\theta_\alpha = \gamma_{\alpha L}^2 - \gamma_{\beta L}^2 + \gamma_{\beta L}^2\cos^2\theta_\beta$$

$$2\gamma_{\alpha\beta}\gamma_{\beta L}\cos\theta_\beta = \gamma_{\alpha\beta}^2 + \gamma_{\beta L}^2 - \gamma_{\alpha L}^2$$

$$\cos\theta_\beta = \frac{\gamma_{\beta L}^2 - \gamma_{\alpha L}^2 + \gamma_{\alpha\beta}^2}{2\gamma_{\alpha\beta}\gamma_{\beta L}}$$

$$\therefore r_\alpha = \frac{S_\alpha}{2\cos\theta_\alpha} = \frac{S_\alpha\gamma_{\alpha\beta}\gamma_{\alpha L}}{\gamma_{\alpha L}^2 - \gamma_{\beta L}^2 + \gamma_{\alpha\beta}^2} \qquad r_\beta = \frac{S_\beta}{2\cos\theta_\beta} = \frac{S_\beta\gamma_{\alpha\beta}\gamma_{\beta L}}{\gamma_{\beta L}^2 - \gamma_{\alpha L}^2 + \gamma_{\alpha\beta}^2}$$

(c) $$\Delta G_{capillary} = \Sigma V_i \frac{dA_i}{dn_i} = \Sigma \gamma_i \frac{h\,dl}{(S_i h\,dl / V_{m,i})} = \Sigma \gamma_i \frac{V_{m,i}}{S_i} = \gamma_{\alpha\beta}\frac{V_{m,\alpha}}{S_\alpha} + \gamma_{\alpha\beta}\frac{V_{m,\beta}}{S_\beta}$$

$V_{m,\alpha} = \frac{S_\alpha}{S}V_m^L$, $V_{m,\beta} = \frac{S_\beta}{S}V_m^L$ 이면,

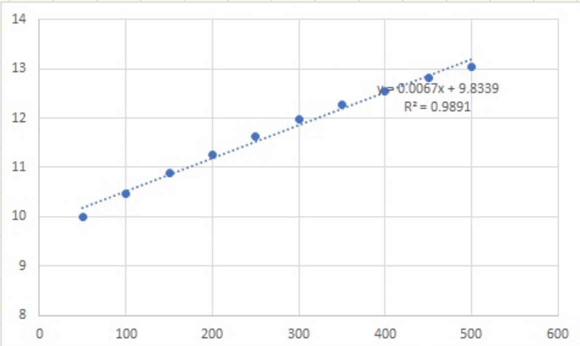
$$\therefore \Delta G_{capillary} = \frac{2\gamma_{\alpha\beta}}{S}V_m^L = \Delta G_{IF}(S)$$

2. $\overline{R} = kt^n$

a) $\ln \overline{R} = n \ln t + \ln k$

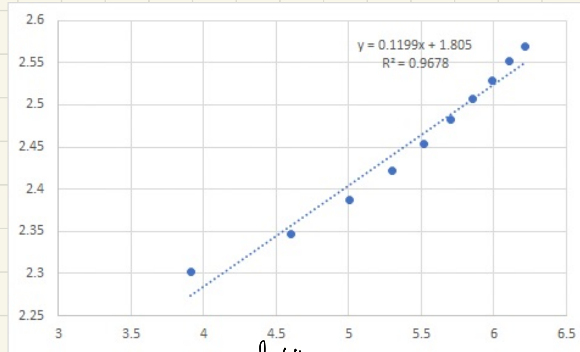
T= 10 , # step = 500.

$\overline{R}$



step

$\overline{R}$



ln(# step)

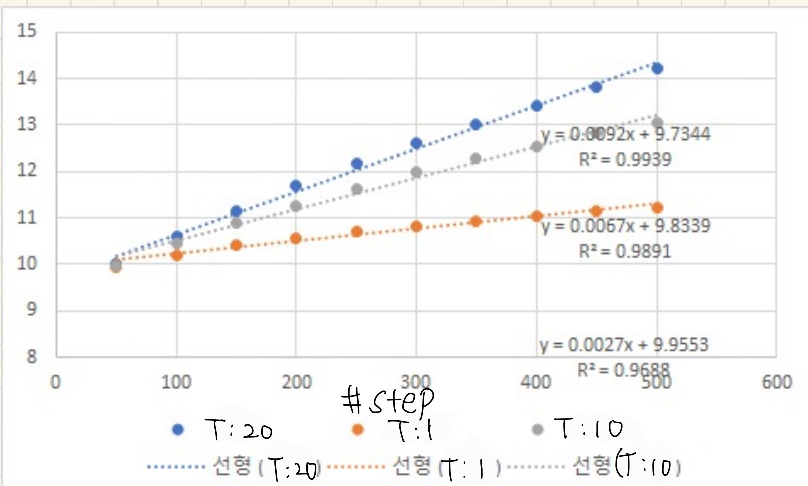
$\Downarrow$

$n \approx 0.1$

time이 증가하면 average grain size도 증가한다.

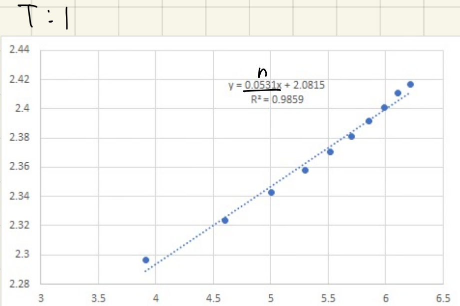
(b)

$\overline{R}$



T: 1

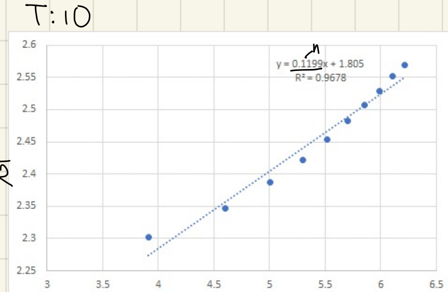
$\ln \overline{R}$



ln (# step)

T: 10

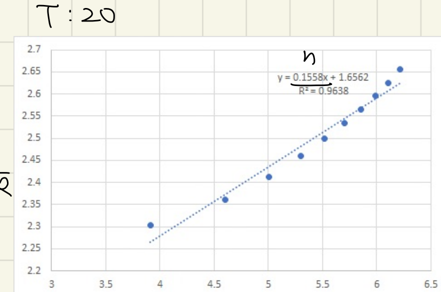
$\ln \overline{R}$



ln (# step)

T: 20

$\ln \overline{R}$



ln (# step)

$\Rightarrow$ T가 증가할수록 n의 값이 증가

(T가 증가하면 grain growth 속도가 증가한다)

$\overline{R} = kt^n = k_0 e^{-Q/RT} t^n \qquad \ln \overline{R} = \ln k_0 - \frac{Q}{RT} + n \ln t$

T: 1 $\Rightarrow$: $\ln k_0 - \frac{Q}{RT} \approx 2.0815$

T: 10 $\Rightarrow$ $\ln k_0 - \frac{Q}{RT} \approx 1.805$

T: 20 $\Rightarrow$ $\ln k_0 - \frac{Q}{RT} \approx 1.6562$

T가 증가할수록 Q↑

(Q 일정하면 T↑ 때 $\ln k_0 - \frac{Q}{RT}$ 값 증가해야 함)