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$$\sum_{i=1}^{n-1} X_i^{\phi} X_n^{\beta} = \sum_{j=1}^{n-1} X_j^{\beta} X_n^{\phi} e^{-\Delta G_j^{seg}/RT}$$

$$\downarrow$$

$$\sum_{i=1}^{n-1} \frac{X_i^{\phi}}{X_n^{\phi}} = \sum_{j=1}^{n-1} \frac{X_j^{\beta}}{X_n^{\beta}} \exp\left(-\frac{\Delta G_j^{seg}}{RT}\right) \quad \text{I}$$

Since $\sum_{i=1}^{n-1} X_i^{\phi} = X_1^{\phi} + \cdots + X_{n-1}^{\phi}$ and $X_1^{\phi} + \cdots + X_n^{\phi} = 1$

then $\sum_{i=1}^{n-1} X_i^{\phi} = 1 - X_n^{\phi}$ II

by plugging II in I $\longrightarrow \frac{1 - X_n^{\phi}}{X_n^{\phi}} = \sum_{j=1}^{n-1} \frac{X_j^{\beta}}{X_n^{\beta}} \exp\left(-\frac{\Delta G_j^{seg}}{RT}\right)$

then $\frac{1}{X_n^{\phi}} = \sum_{j=1}^{n} \frac{X_j^{\beta}}{X_n^{\beta}} \exp\left(-\frac{\Delta G_j^{seg}}{RT}\right) + 1$ (III) , $L = \frac{X_n^{\beta}}{X_n^{\beta}}$ (IV)

by plugging IV in III we have

$$X_n^{\phi} = \frac{X_n^{\beta}}{\sum_{j=1}^{n-1} X_j^{\beta} \exp\left(-\frac{\Delta G_j^{seg}}{RT}\right) + X_n^{\beta}} = \frac{X_n^{\beta}}{\sum_{j=1}^{n-1} X_j^{\beta} \exp\left(-\frac{\Delta G_j^{seg}}{RT}\right) + 1 - \sum_{j=1}^{n-1} X_j^{\beta}}$$

then $X_n^{\phi} = \frac{X_n^{\beta}}{\sum_{j=1}^{n-1} X_j^{\beta} \left(\exp\left(-\frac{\Delta G_j^{seg}}{RT}\right) - 1\right) + 1}$ (V)

we know

$$X_n^{\beta} = X_i^{\beta} \exp\left(-\Delta G_i^{seg}/RT\right) \cdot X_n^{\phi}/X_i^{\phi} \quad \text{(VI)}$$

by plugging VI in V we have

$$\cancel{X_n^{\phi}} X_i^{\phi} = \frac{\cancel{X_n^{\phi}} X_i^{\beta} \exp(-\Delta G_i^{seg}/RT)}{\sum_{j}^{n-1} X_j^{\beta}(\exp(-\frac{\Delta G_j^{seg}}{RT}) - 1) + 1}$$

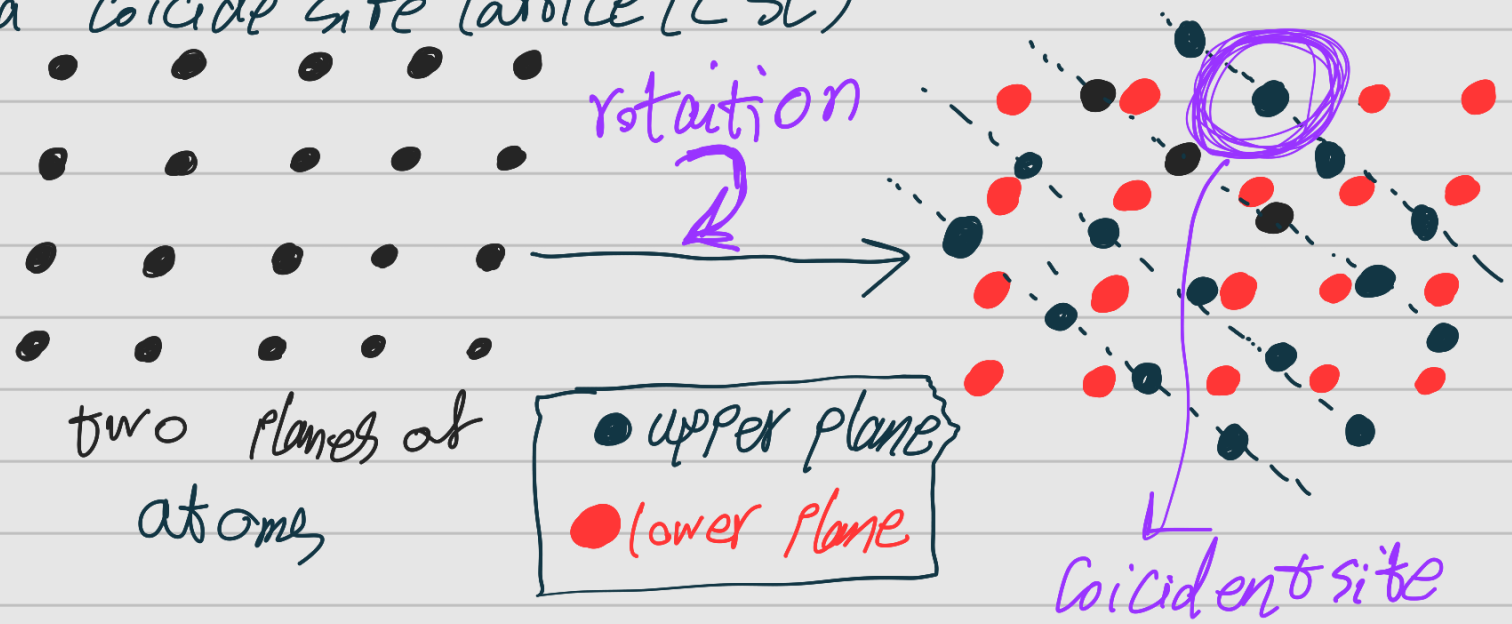
then we have

$$X_i^{\phi} = \frac{X_i^{\beta} \exp(-\Delta G_i^{seg}/RT)}{\sum_{j=1}^{n-1} X_j^{\beta}(\exp(-\Delta G_j^{seg}/RT) - 1) + 1}$$

equation successfully proved ✓

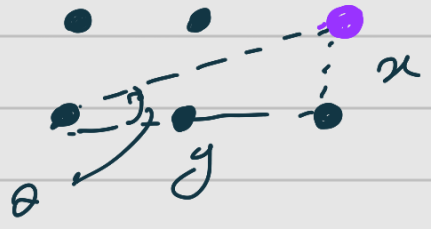
the CSL is a geometrical construction based on the geometry of tha lattice

if two grains of same crystal are placed on top of each other each atom has its own position. Now if the upper plane start to rotate at different angles and some atomes sit on top of each other that means a finite number of lattice site coicides between two lattices. then one can define a coicide site lattice (CSL)



the reason that CSL is important is that when a boundry that contains a high a number of lattice points in a CSL is expected to have good atomic fit and thus low energy.

the angle of rotation can be determined from the lattice geometry because of discrete nature of lattice based on coordinates of coicident site.



$\theta = 2\tan^{-1}(y/x)$ ☆

y should be on same primary position of atom in coicide site befor rotation.