Problem 1

2022년 9월 22일 목요일 오후 2:33

2.1 An monatomic ideal gas at 300 K has a volume of 15 liters at a pressure of 15 atm. Calculate
a. The final volume of the system
b. The work done by the system
c. The heat entering or leaving the system
d. The change in the internal energy
e. The change in the enthalpy when the gas undergoes
   i. A reversible isothermal expansion to a pressure of 10 atm
   ii. A reversible adiabatic expansion to a pressure of 10 atm
The constant-volume molar heat capacity of the gas, c_v, has the value 1.5 R.

$P_1 = 15\,atm,\ V_1 = 15\,L,\ T_1 = 300\,K,\ n = \frac{15 \times 15}{0.08206 \times 300} = 9.14$

i) isothermal $P_2 = 10\,atm$

a. $P_1V_1 = P_2V_2$

$15\,atm \times 15\,L = 10\,atm \times V_2$

$V_2 = 22.5\,L$

b. $dU = \delta q - \delta w$

$dU = C_v dT = 0$

$q = w = \int P dV = nRT \ln\left(\frac{V_2}{V_1}\right) = 9.14 \times 8.31 \times 300 \times \ln\left(\frac{22.5}{15}\right) = 9244\,J$

c. $q = w = 9244\,J$

d. $\Delta U = 0 \quad (\because \Delta T = 0)$

e. $\Delta H = 0 \quad (\because \Delta T = 0)$

ii) adiabatic $P_2 = 10\,atm$

a. $C_v = 1.5R,\ C_p = 2.5R$

$\left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)^{\frac{C_p}{C_v}} \rightarrow \frac{10\,atm}{15\,atm} = \left(\frac{15\,L}{V_2}\right)^{\frac{2.5R}{1.5R}}$

$V_2 = 19.2\,L$

b. $T_2 = \frac{P_2V_2}{nR} = \frac{10 \times 19.2}{9.14 \times 0.082} = 255\,K$

$w = -\Delta U = -nC_v(T_2 - T_1) = -9.14 \times 1.5 \times 8.31 \times (255 - 300) = 5130\,J$

c. $q = 0$

d. $\Delta U = -w = -5130\,J$

e. $\Delta H = nC_p(T_2 - T_1) = 9.14 \times 2.5 \times 8.31 \times (255 - 300) = -8549\,J$

2.5 One mole of N_2 gas is contained at 273 K and a pressure of 1 atm. The addition of 3000 J of heat to the gas at constant pressure causes 832 J of work to be done during the expansion. Calculate
a. The final state of the gas
b. The values of ΔU and ΔH for the change of state
c. The values of c_v and c_p for N_2
Assume that nitrogen behaves as an ideal gas, and that the change of state is conducted reversibly.

N_2 1mole

$T_1 = 273\,K,\ P_1 = 1\,atm \xrightarrow{q = 3000\,J} P_2 = 1\,atm,\ W = 832\,J$

a. $V_1 = \frac{nRT}{P_1} = \frac{1 \times 0.082 \times 273}{1} = 22.4\,L$

$w = P(V_2 - V_1) = 1 \cdot (V_2 - 22.4) \times 101.32 = 832 \quad \therefore V_2 = 30.61\,L$

$T_2 = \frac{P_2V_2}{nR} = \frac{1 \times 30.61}{1 \times 0.082} = 373\,K$

b.c. $q = w + \Delta U$

$\therefore \Delta U = 3000\,J - 832\,J = 2168\,J$

$nC_v\Delta T = 1\,mole \times C_v \times 100\,K = 2168\,J \quad \therefore C_v = 21.68\,J/mole\cdot K,\ C_p = 30\,J/mole\cdot K$

$\Delta H = nC_p\Delta T = (21.68 + 8.3144) \times 100 = 30\,J$

3.2 One mole of a monatomic ideal gas is subjected to the following sequence of steps:
a. Starting at 300 K and 10 atm, the gas expands freely into a vacuum to triple its volume.
b. The gas is next heated reversibly to 400 K at constant volume.
c. The gas is reversibly expanded at constant temperature until its volume is again tripled.
d. The gas is finally reversibly cooled to 300 K at constant pressure.
Calculate the values of q and w and the changes in U, H, and S.

① 1mole, $T_1 = 300\,K$, $P_1 = 10\,atm$, $V_1 = \frac{300 \times 0.082}{10} = 2.46\,L$

② $V_2 = 3V_1 = 7.38\,L$, $P_2 = \frac{1}{3}P_1 = 3.33\,atm$, $T_2 = 300\,K$

③ $V_3 = 7.38\,L$, $T_3 = 400\,K$, $P_3 = \frac{nRT_3}{V_3} = \frac{400 \times 0.082}{7.38} = 4.44\,atm$

④ $V_4 = 3V_3 = 22.14\,L$, $T_4 = 400\,K$, $P_4 = \frac{1}{3}P_3 = 1.48\,atm$

⑤ $V_5 = \frac{nRT_5}{P_5} = \frac{300 \times 0.082}{1.48} = 16.62\,L$, $T_5 = 300\,K$, $P_5 = 1.48\,atm$

①→② : $q = w = nRT \ln\frac{V_2}{V_1} = 300 \times 8.3144 \times \ln 3 = 2740.29\,J$

$\Delta U = \Delta H = 0$

$\Delta S = \frac{\Delta Q}{T} = \frac{2740.29\,J}{300\,K} = 9.13\,J/K$

②→③ : $q = \Delta U = \int C_v dT = 1.5 \times 8.3144 \times (400 - 300) = 1247.16\,J$

$w = 0$

$\Delta H = 2.5 \times 8.3144 \times 100 = 2078.6\,J$

$\Delta S = \frac{\Delta Q}{T} = C_v \ln\frac{T_3}{T_2} = 1.5 \times 8.3144 \times \ln\frac{400}{300} = 3.59\,J/K$

③→④ : $q = w = nRT \ln\frac{V_4}{V_3} = 8.3144 \times 400 \times \ln 3 = 3653.72\,J$

$\Delta U = \Delta H = 0$

$\Delta S = \frac{\Delta Q}{T} = \frac{3653.72\,J}{400\,K} = 9.134\,J/K$

④→⑤ : $w = P(V_5 - V_4) = 1.48 \times (16.62 - 22.14) \times 101.32 = -827.74\,J$

$\Delta U = C_v(T_5 - T_4) = 1.5 \times 8.3144 \times (300 - 400) = -1247.16\,J$

$\Delta H = C_p(T_5 - T_4) = 2.5 \times 8.3144 \times (300 - 400) = -2078.6\,J$

$q = w + \Delta U = -827.74 - 1247.16 = -2074.9\,J$

$\Delta S = C_p \ln\frac{T_5}{T_4} = 2.5 \times 8.3144 \times \ln\frac{3}{4} = -5.980\,J/K$

$\Delta U = 0$
$\Delta H = 0$
$w = 5563$
$q = 5563$
$\Delta S = 15.88$

2.3 The initial state of a quantity of monatomic ideal gas is P = 1 atm, V = 1 liter, and T = 373 K. The gas is isothermally expanded to a volume of 2 liters and is then cooled at constant pressure to the volume V. This volume is such that a reversible adiabatic compression to a pressure of 1 atm returns the system to its initial state. All of the changes of state are conducted reversibly. Calculate the value of V and the total work done on or by the gas.

① $P_1 = 1\,atm,\ V_1 = 1\,L,\ T_1 = 373\,K$ $\xrightarrow{\text{isothermally}}$ ② $P_2 = \frac{1}{2}\,atm,\ V_2 = 2\,L,\ T_2 = 373\,K$ $\xrightarrow{\text{constant pressure}}$ ③ $P_3 = \frac{1}{2}\,atm,\ V_3 = V$ $\xrightarrow{\text{adiabatic}}$ ① $P_1 = 1\,atm,\ V_1 = 1\,L,\ T_1 = 373\,K$

①→② : $P_1V_1 = P_2V_2 \quad \therefore P_2 = \frac{1}{2}\,atm$

$n = \frac{P_1V_1}{RT_1} = \frac{1\,atm \times 1\,L}{0.082\,atm\cdot L/mole\cdot K \times 373\,K} = 0.327$ moles

③→① : $P_3V_3^{\frac{5}{3}} = P_1V_1^{\frac{5}{3}} \quad \therefore V_3 = 1.52\,L$ 즉 V는 1.52L

W계산 ①→② : $w = \int P dV = \int \frac{nRT}{V} dV = nRT \ln\frac{V_2}{V_1} = 0.327\,mole \times 8.3144\,\frac{J}{mole\cdot K} \times 373\,K \times \ln\frac{2L}{1L} = 70.3\,J$

②→③ : $w = P(V_3 - V_2) = \frac{1}{2}\,atm \times (1.52 - 2)\,L = -0.24\,L\cdot atm = -24.5\,J \quad (1\,L\cdot atm = 101.32\,J)$

③→① : $w = -\Delta U = -nC_v\Delta T = -0.0327\,mole \times 1.5 \times 8.3174\,\frac{J}{mole\cdot K} \times (373 - 282)\,K = -37.1\,J$

$\left(T_3 = \frac{P_3V_3}{nR} = \frac{\frac{1}{2} \times 1.52}{0.0327 \times 0.082} = 282\,K \right)$

$W_{TOTAL} = 70.3\,J - 24.5\,J - 37.1\,J = 8.7\,J$

3.1 The initial state of 1 mole of a monatomic ideal gas is P = 10 atm and T = 300 K. Calculate the change in the entropy of the gas for
a. An isothermal decrease in the pressure to 5 atm
b. A reversible adiabatic expansion to a pressure of 5 atm
c. A constant-volume decrease in the pressure to 5 atm

1mole, $P_1 = 10\,atm,\ T_1 = 300\,K$

$V_1 = \frac{nRT}{P_1} = \frac{0.082 \times 300}{10} = 2.462\,L$

a. $P_2 = 5\,atm.\ P_1V_1 = P_2V_2 \quad \therefore V_2 = 4.924\,L$

$\Delta S = \frac{\Delta Q}{T} = \frac{1}{T}\int P dV = \frac{1}{T} nRT \ln\frac{V_2}{V_1} = 8.3144 \times \ln 2 = 5.76\,J/K$

b. $q = 0 \quad \therefore \Delta S = 0$

c. $V_2 = 2.462\,L.\ P_2 = 5\,atm. \quad \therefore T_2 = \frac{P_2V_2}{nR} = 150\,K$

$\Delta S = \frac{\Delta Q}{T} = \frac{\Delta U}{T} = \int \frac{C_v}{T} dT = C_v \ln\frac{T_2}{T_1} = 1.5 \times 8.3144 \times \ln\frac{1}{2} = -8.65\,J/K$

3.3 One mole of a monatomic ideal gas undergoes a reversible expansion at constant pressure, during which the entropy of the gas increases by 14.41 J/K and the gas absorbs 6236 J of thermal energy. Calculate the initial and final temperatures of the gas. One mole of a second monatomic ideal gas undergoes a reversible isothermal expansion, during which it doubles its volume, performs 1729 J of work, and increases its entropy by 5.763 J/K. Calculate the temperature at which the expansion was conducted.

① 등압. 1mole

$\Delta S = 14.41\,J/K.\ q = 6236\,J.\ T_i, T_f = ?$

$\Delta S = C_p \ln\frac{T_f}{T_i} = 14.41,\ q = C_p(T_f - T_i) = 6236$

$\frac{T_f}{T_i} = 2,\ T_f - T_i = 300 \quad \therefore T_f = 600\,K,\ T_i = 300\,K$

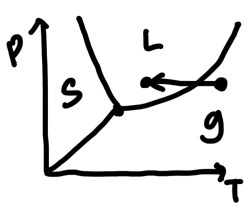
② 등온. 1mole

$V \rightarrow 2V,\ w = 1729\,J,\ \Delta S = 5.763\,J/K$

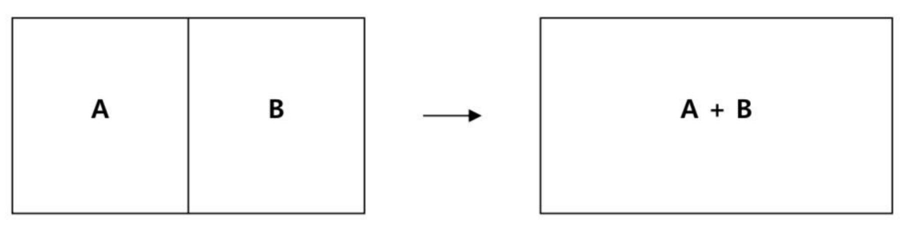
$w = q = 1729\,J.\ \Delta S = \frac{q}{T} \quad \therefore T = \frac{1729}{5.763} = 300\,K$

7. 늦가을 자동차를 운전하면 유리창에 김 서림이 문제가 된다. 자동차 유리창에 김이 서리는 이유를 H_2O의 PT diagram을 이용하여 과학적으로 설명하시오. 이를 제거하기 위해 냉난방 장치를 이용할 경우 창 쪽으로 더운 공기가 나오게 하는 것이 현명한 가, 아니면 에어컨 바람이 나오게 하는 것이 현명한 가? 근거를 대고 설명하시오.

김은 작은 물방울을 말한다. 외부와 내부 공기의 온도 차이 때문에 습기가 액체로 응결되면서 생기는 것이다. 다시말해, 바깥의 따뜻한 수증기가 열을 빼앗기면서 액화되고 물방울이 맺히게 된다. 김서림을 방지하기 위해서는 차량 내부와 외부의 온도 차를 없애야 하고 창 쪽에 에어컨 바람을 나오게 함으로서 외부. 내부의 표면온도를 맞출수 있다



8. 그림 왼쪽과 같이 분리되어 있던 두 종류의 gas 입자들은 칸막이를 제거할 경우 서로 섞여 균일한 혼합체를 이룬다. 각 gas 입자들은 자신들이 섞여 있어야 할 운명이라는 것을 미리 알고 있었을까? (서로 섞여야 한다는 어떤 force 같은 것을 느끼게 되는 걸까?) 이 문제에 대한 견해를 밝히시오.



칸막이를 제거하기 전에도 각 기체 A와 B는 활발하게 운동하고 있었을 것이다 칸막이 제거 후 기체 A와 B는 입자가 움직일 수 있는 범위가 넓어져 각자 독립적으로 공간 전체에 고루 퍼져있을 뿐 섞일 운명임을 알거나 섞여야 한다는 force를 느끼지는 않았을 것이다.