

2.1 An monatomic ideal gas at 300 K has a volume of 15 liters at a pressure of 15 atm. Calculate

- The final volume of the system
- The work done by the system
- The heat entering or leaving the system
- The change in the internal energy
- The change in the enthalpy when the gas undergoes
 - A reversible isothermal expansion to a pressure of 10 atm
 - A reversible adiabatic expansion to a pressure of 10 atm

The constant-volume molar heat capacity of the gas, c_v , has the value $1.5 R$.

$$PV = nRT, \quad n = \frac{PV}{RT} = \frac{(15 \text{ atm}) \times (15 \text{ L})}{(0.08206 \text{ atm} \cdot \text{L} / \text{K} \cdot \text{mol}) \times (300 \text{ K})} = 9.14 \text{ mol}$$

$$(i) \text{ isothermal: } \Delta U = 0, \quad q = w = \int_{\alpha}^{\beta} P dV = \int_{\alpha}^{\beta} nRT \cdot \frac{1}{V} dV$$

$$P_{\alpha} V_{\alpha} = P_{\beta} V_{\beta} \quad \Rightarrow \quad nRT \left[\ln V \right]_{\alpha}^{\beta} = 9.14 \times 8.314 \times 300 \times \ln \frac{22.5}{15} = 9.243 \text{ kJ}$$

$$V_{\beta} = \frac{P_{\alpha} V_{\alpha}}{P_{\beta}} = \frac{(15 \text{ atm}) (15 \text{ L})}{(10 \text{ atm})} = 22.5 \text{ L}$$

$$(a) V_{\beta} = 22.5 \text{ L} \quad (b) w = 9.243 \text{ kJ} \quad (c) q = 9.243 \text{ kJ} \quad (d) \Delta U = 0 \quad (e) \Delta H = 0$$

$$(ii) \text{ adiabatic: } q = 0, \quad \Delta U = -w = \int_{\alpha}^{\beta} P dV = \int_{\alpha}^{\beta} n c_v dT = 9.14 \times \frac{3}{2} R \times (255 - 300) = -5.129 \text{ kJ}$$

$$c_p = \frac{5}{2} R, \quad c_v = \frac{3}{2} R \Rightarrow \gamma = \frac{5}{3}$$

$$P_{\alpha} V_{\alpha}^{\gamma} = P_{\beta} V_{\beta}^{\gamma} \rightarrow V_{\beta} = \left(\frac{P_{\alpha} V_{\alpha}}{P_{\beta}} \right)^{\frac{1}{\gamma}} = 19.13 \text{ L} \quad T_{\beta} = \frac{P_{\beta} V_{\beta}}{nR} = \frac{10 \times 19.13}{9.14 \times 0.08206} = 255 \text{ K} \quad \Delta H = \int n c_p dT = 9.14 \times \frac{5}{2} R \times (255 - 300) = 8.579 \text{ kJ}$$

$$(a) V_{\beta} = 19.13 \text{ L} \quad (b) w = 5.129 \text{ kJ} \quad (c) q = 0 \quad (d) \Delta U = -5.129 \text{ kJ} \quad (e) \Delta H = 8.579 \text{ kJ}$$

2.3 The initial state of a quantity of monatomic ideal gas is $P = 1 \text{ atm}$, $V = 1 \text{ liter}$, and $T = 373 \text{ K}$. The gas is isothermally expanded to a volume of 2 liters and is then cooled at constant pressure to the volume V . This volume is such that a reversible adiabatic compression to a pressure of 1 atm returns the system to its initial state. All of the changes of state are conducted reversibly. Calculate the value of V and the total work done on or by the gas.

$a \rightarrow b \rightarrow c$

$$\bullet \text{ isothermal: } \Delta U = 0, \quad q = w = \int_{\alpha}^{\beta} P dV = nRT \left(\ln \frac{2}{1} \right) = 0.0327 \times 8.314 \times 373 \times \ln 2 = 70.3 \text{ J}$$

$$n = \frac{PV}{RT} = \frac{1 \times 1}{0.08206 \times 373} = 0.0327 \text{ mol} \quad / \quad P_b = \frac{P_{\alpha} V_{\alpha}}{V_b} = \frac{1 \times 1}{2} = \frac{1}{2} \text{ atm}$$

$$\bullet C_v: \quad w = P_c (V_c - V_b) = \frac{1}{2} \times (1.516 - 2) = -0.242 \text{ L} \cdot \text{atm} \rightarrow (-0.242 \text{ L} \cdot \text{atm}) \times \frac{(8.314 \text{ J/mol} \cdot \text{K})}{(0.08206 \text{ L} \cdot \text{atm/mol} \cdot \text{K})} = -24.5 \text{ J}$$

$$V_c = \left(\frac{P_{\alpha} V_{\alpha}^{\gamma}}{P_c} \right)^{\frac{1}{\gamma}} = \left(\frac{1 \times 1^{\frac{5}{3}}}{\frac{1}{2}} \right)^{\frac{3}{5}} = 1.516 \text{ L}$$

$$\bullet \text{ adiabatic: } q = 0, \quad \Delta U = -w = - \int_{\alpha}^{\beta} n c_v dT = -0.0327 \times \frac{3}{2} R \times (373 - 282) = -37.1 \text{ J} \quad \Rightarrow 70.3 - 24.5 - 37.1 = \boxed{8.7 \text{ J}}$$

$$T_c = \frac{P_c V_c}{nR} = \frac{\frac{1}{2} \times 1.516}{0.0327 \times 0.08206} = 282 \text{ K}$$

2.5 One mole of N_2 gas is contained at 273 K and a pressure of 1 atm. The addition of 3000 J of heat to the gas at constant pressure causes 832 J of work to be done during the expansion. Calculate

- The final state of the gas
- The values of ΔU and ΔH for the change of state
- The values of c_v and c_p for N_2

Assume that nitrogen behaves as an ideal gas, and that the change of state is conducted reversibly.

$$n=1, T_1=273K, P_1=1\text{ atm}$$

$$V_1 = \frac{nRT_1}{P_1} = \frac{1 \times 0.08206 \times 273}{1} = 22.4\text{ L}, \quad P(V_2 - V_1) = 832\text{ J} \rightarrow (1\text{ atm}) \times (V_2 - V_1)\text{ L} = (832\text{ J}) \times \frac{(0.08206\text{ L}\cdot\text{atm}/\text{mol}\cdot\text{K})}{(8.314\text{ J}/\text{mol}\cdot\text{K})}$$

$$T_2 = \frac{P_2 V_2}{nR} = \frac{1 \times 30.61}{1 \times 0.08206} = 373\text{ K} \quad \Rightarrow V_2 = 30.61\text{ L}$$

$$(a) T_2 = 373\text{ K}, V_2 = 30.61\text{ L}, n=1, P_2 = 1\text{ atm}$$

$$(b) \Delta U = q - w = 3000\text{ J} - 832\text{ J} = 2168\text{ J}$$

$$\Delta H = q_p = 3000\text{ J}$$

$$(c) c_v(T_2 - T_1) = 2168\text{ J} \rightarrow c_v = \frac{2168}{(373 - 273)} = 21.7\text{ J}/\text{mol}\cdot\text{K}$$

$$c_p(T_2 - T_1) = 3000\text{ J} \rightarrow c_p = \frac{3000}{(373 - 273)} = 30\text{ J}/\text{mol}\cdot\text{K}$$

3.1 The initial state of 1 mole of a monatomic ideal gas is $P = 10 \text{ atm}$ and $T = 300 \text{ K}$.

Calculate the change in the entropy of the gas for

- An isothermal decrease in the pressure to 5 atm
- A reversible adiabatic expansion to a pressure of 5 atm
- A constant-volume decrease in the pressure to 5 atm

$$3.1) n = 1 \text{ mol}, P_1 = 10 \text{ atm}, T_1 = 300 \text{ K}$$

$$(a) \text{ isothermal: } \Delta U = 0, q = w, P_1 V_1 = P_2 V_2, V_2 = 2V_1$$

$$\Delta S = \frac{q}{T}, q = w = \int P dV = nRT \int_{V_1}^{V_2} \frac{1}{V} dV$$
$$= 1 \times 8.314 \times 300 \times \ln 2$$
$$= 1729 \text{ J}$$

$$\therefore \Delta S = \frac{1729}{300 \text{ K}} = 5.76 \text{ J/K}$$

$$(b) \text{ reversible adiabatic, } \Delta S = 0.$$

$$(c) C_V: w = 0, \Delta U = q = n \int_{T_1}^{T_2} C_V dT$$

$$T_2 = \frac{P_2 V_2}{nR} = \frac{5 \times 2.462}{1 \times 0.08206} = 150 \text{ K}$$

$$V_1 = \frac{nRT_1}{P_1} = \frac{1 \times 0.08206 \times 300}{10} = 2.462 \text{ L} = V_2$$

$$\Delta S = \int \frac{dq}{T} = n \int_{T_1}^{T_2} \frac{C_V}{T} dT = n \times \frac{3}{2} R \times \ln \left(\frac{300}{150} \right) = 8.64 \text{ J/K}$$

3.2 One mole of a monatomic ideal gas is subjected to the following sequence of steps:

- Starting at 300 K and 10 atm, the gas expands freely into a vacuum to triple its volume.
 - The gas is next heated reversibly to 400 K at constant volume.
 - The gas is reversibly expanded at constant temperature until its volume is again tripled.
 - The gas is finally reversibly cooled to 300 K at constant pressure.
- Calculate the values of q and w and the changes in U , H , and S .

3.2) $n = 1 \text{ mol}$

(a) 자유 팽창 : $w = q = \Delta U = \Delta H = 0$

$T_1 = 300 \text{ K}$, $P_1 = 10 \text{ atm}$, $V_2 = 3V_1 \rightarrow P_2 = 1 \text{ atm}$

$\Delta S = nR \ln \left(\frac{V_2}{V_1} \right) = 1 \times 8.314 \times \ln 3 = 9.134 \text{ J/K}$

(free expansion)

(b) constant V : $w = 0$, $\Delta U = q_v = q$, $V_2 = V_1 = 3V_1$

$q = C_v \times (T_3 - T_2) = \frac{3}{2}R \times (400 - 300) = 1247 \text{ J}$

$\Delta H = C_p \times (T_3 - T_2) = \frac{5}{2}R \times (400 - 300) = 2079 \text{ J}$

$\Delta S = \int \frac{dq}{T} = \int \frac{nC_v dT}{T} = 1 \times \frac{3}{2}R \times \ln \left(\frac{400}{300} \right) = 3.59 \text{ J/K}$

(c) isothermal : $\Delta U = \Delta H = 0$, $w = q$, $V_4 = 3V_3 = 9V_1$

$q = w = nRT \int \frac{1}{V} dV = 1 \times 8.314 \times 400 \times \ln 3 = 3654 \text{ J}$

$\Delta S = \frac{q}{T} = \frac{3654}{400} = 9.13 \text{ J/K}$

(d) Constant P : $T_5 = 300 \text{ K}$, $\Delta U = q - w$, $\Delta H = q_p = q$

$q = C_p (T_5 - T_4) = \frac{5}{2}R \times (300 - 400) = -2079 \text{ J} = \Delta H$

$\Delta U = C_v (T_5 - T_4) = \frac{3}{2}R \times (300 - 400) = -1247 \text{ J}$

$w = q - \Delta U = (-2079) - (-1247) = -832 \text{ J}$

$\Delta S = \int \frac{dq}{T} = \int \frac{nC_p dT}{T} = 1 \times \frac{5}{2}R \times \ln \left(\frac{300}{400} \right) = -5.98 \text{ J/K}$

Total : $q = 2822 \text{ J}$, $w = 2822 \text{ J}$, $\Delta U = 0$, $\Delta H = 0$

$\Delta S = 15.88 \text{ J/K}$

- 3.3 One mole of a monatomic ideal gas undergoes a reversible expansion at constant pressure, during which the entropy of the gas increases by 14.41 J/K and the gas absorbs 6236 J of thermal energy. Calculate the initial and final temperatures of the gas. One mole of a second monatomic ideal gas undergoes a reversible isothermal expansion, during which it doubles its volume, performs 1729 J of work, and increases its entropy by 5.763 J/K . Calculate the temperature at which the expansion was conducted.

3.3). Constant P: $q = 6236 \text{ J} = \Delta H = q_p$, $\Delta S = 14.41 \text{ J/K}$.

$$\Delta S = \int \frac{dq}{T} = \int n C_p \cdot \frac{1}{T} dT = \frac{5}{2} R \times \ln \left(\frac{T_2}{T_1} \right) = 14.41 \text{ J/K}$$

$$\frac{T_2}{T_1} = e^{14.41 \times \frac{2}{5R}} = 2 \rightarrow T_2 = 2T_1$$

$$q_p = C_p (T_2 - T_1) \Rightarrow T_1 = \frac{2}{5R} \times 6236 = 300 \text{ K}$$

$$T_1 = 300 \text{ K}, T_2 = 600 \text{ K}$$

isothermal: $\Delta U = 0$, $q = W$, $V_2 = 2V_1$

$$W = nRT \int \frac{1}{V} dV = 1 \times 8.314 \times T \times \ln 2 = 1729$$

$$T = \frac{1729}{8.314 \times \ln 2} = 300 \text{ K}$$

7. 일반적으로 물기름에는 자유히의 내부 압도가 외부 압도보다 높다. 또한 건조한 가을 공기에 비해 차가운 날씨에는 사람들이 단속해 있기때 공기를 H_2O 의 분압 또한 외부보다 높다. 이때 외부의 공기로 인해 차가워진 유체와 수증기가 만나면서 수증기로 H_2O 의 온도가 하락하게 되고 PT diagram 상에서 vapor보다 liquid가 안정한 상태로 H_2O 의 특성이 변화하게 된다. 이는 내·외부의 기압차로 인해 발생하는 현상이므로 하더라도 에어리어를 작음시켜 내·외부의 온도차를 줄이고, 에어리어의 계층 계층을 통해 공기를 H_2O 분압을 낮추는 것이 도움이 될 것이다.

8. gas의 각 입자들은 '자기' 다른 운동을 하며 각 공간 내에 어느정도 균등하게 퍼져 있을 것이다. 이는 입자가 한 곳에 몰려 있을 것이다 균등하게 흩어져 있을 상태일 확률이 훨씬 높기 때문이다. 이때 한곳을 채우면 A, B gas 입자들은 각각 전체 공간에 균등하게 분포하는 방향으로 움직일 것이다. 따라서 gas를 자신의 운동에 따라 전체 공간에서 특정 공간을 차지했을 확률, 어떠한 force를 느끼지는 않았을 것이다. 입자들의 움직임에 따른 결과가 균일한 분포치라는 확률로써 나타난 것이 입자들은 그렇게 될 것이라는 것도 예상하지 않았을 것이다.