HW1

2.1 An monatomic ideal gas at 300 K has a volume of 15 liters at a pressure of 15 atm. Calculate

a. The final volume of the system
b. The work done by the system
c. The heat entering or leaving the system
d. The change in the internal energy
e. The change in the enthalpy when the gas undergoes
   i. A reversible isothermal expansion to a pressure of 10 atm
   ii. A reversible adiabatic expansion to a pressure of 10 atm

The constant-volume molar heat capacity of the gas, c_v, has the value 1.5 R.

$T_1 = 300K$, $V_1 = 15L$, $P_1 = 15 atm$

i) reversible isothermal expansion to a pressure of 10 atm

a. $P_1V_1 = nRT$ $\quad 15 \times 15 = n \times R \times 300$, $\quad n = \frac{15 \times 15}{300 \times 0.08206} = 9.14\ mol$

isothermal $\Rightarrow$ $P_1V_1 = P_2V_2$, $\quad 15 \times 15 = 10 \times V_2$, $\quad V_2 = 22.5\ L$

b. $\because dU = 0 \quad \therefore w = q = nRT \ln\left(\frac{V_2}{V_1}\right) = 9.14 \times 8.3144 \times 300 \times \ln\left(\frac{22.5}{15}\right) = 9244\ J$

c. $q = w = 9244\ J$

d. $\Delta T = 0 \Rightarrow \Delta U = 0$ $\quad$ e. $\Delta T = 0 \Rightarrow \Delta H = 0$

ii) reversible adiabatic expansion to a pressure of 10 atm

a. $P_1V_1^{\gamma} = P_2V_2^{\gamma}$ & $\frac{R}{C_V} = \gamma - 1$ $\quad (C_V = 1.5R)$

$\frac{2}{3} = \gamma - 1$, $\gamma = \frac{5}{3}$

$15 \times 15^{\frac{5}{3}} = 10 \times V_2^{\frac{5}{3}}$, $\quad V_2 = 19.13\ L$

b. $q = 0 \Rightarrow \Delta U = -W$, $\quad W = -\Delta U = -C_V dT = -1.5R(T_2 - T_1)$

T_2 求得: $\quad PV = nRT \rightarrow T_2 = \frac{P_2V_2}{nR} = \frac{10 \times 19.13}{9.14 \times 0.08206} = 255K$

$\Rightarrow -1.5R(255 - 300)$

$= 513J = W$

c. $q = 0$

d. $\Delta U = -W = -513J$ $\quad$ e. $dH = nC_p dT$ & $\frac{C_p}{C_V} = \gamma$

$C_p = \frac{5}{3}C_V = \frac{5}{3} \times \frac{3}{2}R = \frac{5}{2}R$

$\therefore \Delta H = 9.14\ mol \times \frac{5}{2}R(T_2 - T_1)$

$= 9.14 \times \frac{5}{2} \times 8.3144(255 - 300) = -8549\ J$

2.3 The initial state of a quantity of monatomic ideal gas is $P = 1$ atm, $V = 1$ liter, and $T = 373$ K. The gas is isothermally expanded to a volume of 2 liters and is then cooled at constant pressure to the volume V. This volume is such that a reversible adiabatic compression to a pressure of 1 atm returns the system to its initial state. All of the changes of state are conducted reversibly. Calculate the value of V and the total work done on or by the gas.

$P_1 = 1\ atm,\ V_1 = 1L,\ T_1 = 373K$: state ①

isothermally expanded → $V_2 = 2L$: state ②

cooled at constant pressure → V : state ③

i) ① → ② : 등온팽창, $PV = nRT \Rightarrow P_1V_1 = P_2V_2$

$P_2 = \frac{P_1V_1}{V_2} = \frac{1\times1}{2} = \frac{1}{2}$

① ↔ ③ : 단열 과정, $P_1V_1^{\gamma} = P_3V^{\gamma}$ $(P_3 = P_2)$

$1 = P_3V^{\frac{5}{3}}$ $\therefore V = 1.52L$

ii) total W : ① → ② $W = q = nRT_1\ln\left(\frac{V_2}{V_1}\right) = 0.0327\times8.314\times373\ln2 = 70.3J$

$\left(n = \frac{P_1V_1}{RT_1} = 0.0327\ mol\right)$

② → ③ $W = P_2dV = \frac{1}{2}\times(V - V_2) = \frac{1}{2}\times(1.52-2) = -0.24\ L\cdot atm = -24.5J$

③ → ① $-W = \Delta U = nC_vdT$

$\Rightarrow W = -nC_vdT = -0.0327\frac{3}{2}(T_1 - T_3) = -39.1J$

$\therefore W_{total} = 70.3J - 24.5J - 39.1J$

$= 8.7J$ done by gas

2.5 One mole of N_2 gas is contained at 273 K and a pressure of 1 atm. The addition of 3000 J of heat to the gas at constant pressure causes 832 J of work to be done during the expansion. Calculate

a. The final state of the gas
b. The values of ΔU and ΔH for the change of state
c. The values of c_v and c_p for N_2

Assume that nitrogen behaves as an ideal gas, and that the change of state is conducted reversibly.

1 mol N_2 $\frac{273\,K}{T_1}$ $\frac{1\,atm}{P_1}$ ⟹ (3000J = q (등압) 832J = w) final state

a. final state. $PV = nRT$ (P 등압, n = 1) ⟹ $P_1V_1 = R \cdot 273K$

$V_1 = R \cdot 273K = 0.08206 \times 273 = 22.4\,l$

$W = 832J = P_1 dV \times \frac{8.3144\,J/mol\,K}{0.08206\,l\,atm/K}$ · V_1은 알아야 함

$832J = P_1 (V_2 - V_1) \times \frac{8.3144}{0.08206}$, $V_2 = 30.61\,l$

$\Rightarrow V_2 = RT_2$, $T_2 = \frac{30.61}{0.08206} = 373K$

b. $\Delta U = q - w = 3000 - 832 = 2168J = \Delta U$

$\Delta H = q_p = 3000J$

c. $\Delta H = q_p = nC_p dT = C_p(373 - 273)$. $\therefore C_p = \frac{3000}{100} = 30\,J/mol \cdot K = C_p$

$\Delta U = nC_v dT \Rightarrow \frac{2168J}{(373-273)\,K} \frac{1}{n} = C_v = 21.7\,J/mol \cdot K$

3.1 The initial state of 1 mole of a monatomic ideal gas is $P = 10$ atm and $T = 300$ K. Calculate the change in the entropy of the gas for

a. An isothermal decrease in the pressure to 5 atm
b. A reversible adiabatic expansion to a pressure of 5 atm
c. A constant-volume decrease in the pressure to 5 atm

1몰 $P_1 = 10\,atm,\ T_1 = 300k$

$PV = nRT \Rightarrow V_1 = \frac{1 \times 0.08206 \times 300}{10} = 2.462l$

a. $P_2 = 5\,atm,\ T_2 = 300k.\quad \Delta S = R \ln \frac{V_2}{V_1}$

$P_2V_2 = 1 \cdot R \cdot 300,\ V_2 = 4.924l$

$\Delta S = 8.3144 \ln 2 = 5.76\ J/k$

b. 단열과정. $P_2 = 5\,atm$

$q = 0 \Rightarrow \Delta S = \frac{q_{rev}}{T} = 0$

c. 등적 $P_2 = 5\,atm$

$V_2 = 2.462l$

$T_2 = \frac{P_2 \cdot V_2}{1 \cdot R} = 150k$

$\Delta S = C_v \ln \frac{T_2}{T_1} = 1.5 \times 8.3144 \times \ln \frac{1}{2} = -8.65\ J/k$

3.2 One mole of a monatomic ideal gas is subjected to the following sequence of steps:

a. Starting at 300 K and 10 atm, the gas expands freely into a vacuum to triple its volume.
b. The gas is next heated reversibly to 400 K at constant volume.
c. The gas is reversibly expanded at constant temperature until its volume is again tripled.
d. The gas is finally reversibly cooled to 300 K at constant pressure.

Calculate the values of q and w and the changes in U, H, and S.

a. $T_1 = 300K$, ① $P_1 = 10atm$ ⟹ expand ② volume 3배

b. C_V 3 400K 까지 가열

c. 부피 3배 팽창 (등온 과정) ④ d. C_p 3 ⑤ 300K까지 냉각

a. ①→② $PV = nRT$ $10V_1 = RT_1$, $V_1 = \frac{300R}{10} = 2.462l$ $P_2 = \frac{P_1}{3} = 3.33\ atm$

$V_2 = 2.462 \times 3 = 7.386\ L$ $T_2 = 300K$

⇒ ($q = w = \Delta U \overset{??}{=} \Delta H = 0$ $\Delta S = R\ln\frac{V_2}{V_1} = 8.314 \ln 3 = 9.134\ J/K$)

b. ②→③ constant volume $T_3 = 400K$, $V_3 = 7.386\ L$

$PV = nRT$. $\frac{T_2}{P_2} = \frac{T_3}{P_3}$, $P_3 = \frac{T_3 P_2}{P_2} = \frac{4}{3} \times 3.33 = 4.44\ atm$

$\Delta V = 0$ 이므로 → $w = PdV = 0$, $q \overset{\Delta U}{=} C_V \Delta T = C_V(T_3 - T_2) = \frac{3}{2} \times 8.314 \times (400-300)$

$= 1247J$

$\Delta H = C_p(T_3 - T_2) = \frac{5}{3} \times 100 = \frac{500}{3} = 166.6\ J$

$\Delta S = C_p \ln\frac{T_3}{T_2} = \overset{C_V R}{1.5} \times 8.314 \ln\frac{4}{3} = 3.588\ J/K$ ⇒ ($\Delta U = q = 1247J$, $w = 0$, $\Delta H = 166.6J$, $\Delta S = 3.588\ J/K$)

c. ③→④ 부피 3배 늘리기. $V_3 \times 3 = V_4 = 22.158L$, 등온 ⇒ $T_4 = 400K$ $P_4 = \frac{P_3}{3} = 1.481\ atm$

$\Delta U = \Delta H = 0$. $q = w = RT_4 \ln\frac{V_4}{V_3} = 8.314 \times 400 \ln 3 = 3654J$

$\Delta S = \frac{q}{T} = \frac{3654}{T_4} = 9.134\ J/K$ ⇒ $q = w = 3654J$, $\Delta U, \Delta H = 0$, $\Delta S = 9.134\ J/K$

d. (4)→(5) $T_5 = 300k$ $P_5 = 1.481 atm$ $V_5 = \frac{0.08206 \times 300}{1.481} \doteq 16.623\,L$

$\Delta U = C_v (T_5 - T_4) = \frac{3}{2} \times 8.3144 (300 - 400) = -1247J$

$\Delta H = q_p = C_p \Delta T = \frac{5}{2} \times 8.3144 (300 - 400) = -2078J$

$W = q - \Delta U = -2078 + 1247 = -831J$

$\Delta S = C_p \ln \frac{T_5}{T_4} = \frac{5}{2} \times 8.3144 \ln \frac{3}{4} = -5.980 J/k$

⇒ $q = -2078J$, $W = -831J$
$\Delta U = -1247J$, $\Delta H = -2078J$
$\Delta S = -5.980 J/k$

$\Delta U = 0 + 1247 + 0 - 1247 = 0$ $\quad W = 0 + 4354 - 831 = 2823$

⇒ $\Delta H = 0 + 2078 + 0 - 2078 = 0$ $\quad q = 0 + 1497 + 3854 - 2078 = 2823$

$\Delta S = 9.134 + 3.588 + 9.134 - 5.980 = 15.88$

3.3 One mole of a monatomic ideal gas undergoes a reversible expansion at constant pressure, during which the entropy of the gas increases by 14.41 J/K and the gas absorbs 6236 J of thermal energy. Calculate the initial and final temperatures of the gas. One mole of a second monatomic ideal gas undergoes a reversible isothermal expansion, during which it doubles its volume, performs 1729 J of work, and increases its entropy by 5.763 J/K. Calculate the temperature at which the expansion was conducted.

1) 1mol ideal gas, C_p 알고 $\Delta S = 14.41 J/k$ $q = 6236J$ → T_f, T_i 구하기

2) isothermal expansion, 부피 2배, $W = 1729J$, $\Delta S = 5.763 J/k$

T_{final} 구하기

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1) $\Delta S = C_p \ln \frac{T_2}{T_1} = 2.5 \times 8.3144 \ln \frac{T_2}{T_1} = 14.41$

$\frac{T_2}{T_1} = 2$

$q = C_p \Delta T = \frac{5}{2} \times 8.3144 (T_2 - T_1) = 6236$, $T_2 - T_1 = 300$ → $T_2 = 600k$, $T_1 = 300k$

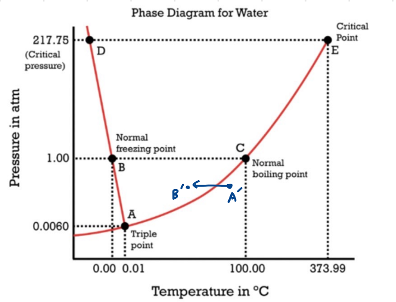
2) isothermal ⇒ $\Delta U = 0$, $q = W = 1729$

$\Delta S = \frac{q}{T} = 5.763$ ∴ $T = 300k$

or

$W = RT \ln \frac{V_2}{V_1} = RT \ln 2 = 1729$ ∴ $T = \frac{1729}{\ln 2 \times 8.3144} = 300k$

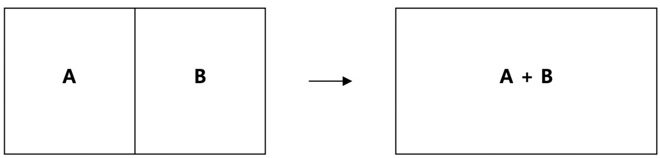
7. 늦가을 자동차를 운전하면 유리창에 김 서림이 문제가 된다. 자동차 유리창에 김이 서리는 이유를 H_2O의 PT diagram을 이용하여 과학적으로 설명하시오. 이를 제거하기 위해 냉난방 장치를 이용할 경우 창 쪽으로 더운 공기가 나오게 하는 것이 현명한 가, 아니면 에어컨 바람이 나오게 하는 것이 현명한 가? 근거를 대고 설명하시오.



겨울철에 외부의 차가운 날씨로 인해 자동차의 창문이 차가워져 있다. 이 때 차 내부의 수증기는 창문보다 온도가 높고 차가운 창문과 접촉하면서 기온이 내려가 (A'→B') 김이 서리게 되는 것이다.

창문에 서린 김을 제거하기 위해서는 내부의 수증기 온도도 낮춰야 한다. 따라서 창쪽으로 에어컨 바람이 나오게 하여 김 서림을 방지한다.

8. 그림 왼쪽과 같이 분리되어 있던 두 종류의 gas 입자들은 칸막이를 제거할 경우 서로 섞여 균일한 혼합체를 이룬다. 각 gas 입자들은 자신들이 섞여 있어야 할 운명이라는 것을 미리 알고 있었을까? (서로 섞여야 한다는 어떤 force 같은 것을 느끼게 되는 걸까?) 이 문제에 대한 견해를 밝히시오.



기체가 존재할 수 있는 부피가 커지면 엔트로피가 증가한다.
칸막이를 제거하면서 기체 A, B가 존재할 수 있는 공간이 커지게 되고 기체들은 가장 안정한 상태가 될 수 있도록 퍼져나가는 운동을 한다.
이 자발적인 반응 과정에서 엔트로피가 증가한다.
서로 섞여야 한다는 force는 엔트로피의 증가인 것 같다.