2.1 An monatomic ideal gas at 300 K has a volume of 15 liters at a pressure of 15 atm. Calculate

a. The final volume of the system
b. The work done by the system
c. The heat entering or leaving the system
d. The change in the internal energy
e. The change in the enthalpy when the gas undergoes
    i. A reversible isothermal expansion to a pressure of 10 atm
    ii. A reversible adiabatic expansion to a pressure of 10 atm

The constant-volume molar heat capacity of the gas, c_v, has the value 1.5 R.

Monoatomic, T=300K, V=15L, P=15atm

#a

I) $\Delta U = 0$, $P = 10\,atm$

$$n = \frac{15 \times 15\,(L\cdot atm)}{0.082\,L\cdot atm/K\cdot mol \times 300K} = 9.15\,mol \qquad V_2 = \frac{0.082 \times 300K \times 9.15\,mol}{10\,atm} = 22.5L$$

(L·atm/K·mol)

II) $q = 0$, $P = 10\,atm$

$$P_1 V_1^{\gamma} = 15 \times 15^{\frac{5}{3}} = P_3 V_3^{\frac{5}{3}} = 10 V_3^{\frac{5}{3}} \Rightarrow V_3 = 19.13L$$

#b

I) $$W = \int P\,dV \Rightarrow -\int_{V_1}^{V_2} \frac{nRT}{V}\,dV = -\int_{15L}^{22.5L} \frac{9.15mol \times 8.314J/K\cdot mol \times 300K}{V}\,dV$$

$$= -9.15mol \times 8.314J/K\cdot mol \times 300K \times \ln\left(\frac{22.5L}{15L}\right)$$

$$= -9253J$$

II) $q = 0$, $\Delta U = W$, $dU = C_v dT$ $\quad (T_1 = 300K)$

$$\Delta U = \int \frac{3}{2} \times 8.314J/K\cdot mol \times 9.15mol\,dT \quad \left(T_2 = \frac{10 \times 19.13}{9.15 \times 0.082} = 255K\right)$$

$$= \frac{3}{2} \times 8.314J/K\cdot mol \times (255 - 300K) \times 9.15mol$$

$$= -5135J$$

#c

I) $\cancel{\Delta U}^{0} = q + w$, $q = -w$, $w = -9253\,J$

$\Rightarrow q = 9253\,J$

II) $q = 0\,J$

#d

I) $\Delta U = 0\,J$

II) $\Delta U = \int C_v\,dT = -5134\,J$

#e

I) $\Delta H = \cancel{\Delta U}^{0} + \Delta(PV) = \Delta(nR\cancel{T})^{0} = 0\,J$

II) $\Delta H = \int C_p\,dT = 9.14\,mol \times \frac{5}{2} \times 8.314\,J/K\cdot mol \times (254K - 300K)$

$= -8448\,J$

2.3 The initial state of a quantity of monatomic ideal gas is $P = 1$ atm, $V = 1$ liter, and $T = 373$ K. The gas is isothermally expanded to a volume of 2 liters and is then cooled at constant pressure to the volume V. This volume is such that a reversible adiabatic compression to a pressure of 1 atm returns the system to its initial state. All of the changes of state are conducted reversibly. Calculate the value of V and the total work done on or by the gas.

STEP 1. $P = 1\text{atm},\ V = 1\text{L},\ T = 373\text{K}$, $n = \frac{1 \times 1}{0.082 \times 373} = 0.033\text{mol}$

$\Delta U = 0,\ V \Rightarrow 1\text{L} \rightarrow 2\text{L}$, $W = -\int P dV = -\int_{1L}^{2L} \frac{0.033\text{mol} \times 8.314\text{J/K·mol} \times 373\text{K}}{V} dV$

$= -70\text{J}$

$P = \frac{0.082\text{L atm/K mol} \times 0.033\text{mol} \times 373\text{K}}{2\text{L}} = 0.5\text{atm}$

STEP 2. $P_1 V_1^{\frac{5}{3}} = P_3 V_3^{\frac{5}{3}} \Rightarrow 1 = 0.5 V_3^{\frac{5}{3}} \Rightarrow V_3 = 1.52\text{L}$

$W = -P\Delta V = -0.5\text{atm}(1.52 - 2) = 24.5\text{J}$

STEP 3. $q = 0,\ W = \Delta U$

$\Delta U = \int C_v dT = 0.033\text{mol} \times \frac{3}{2} \times 8.314\text{J/K·mol}$

$\times (373\text{K} - 282\text{K}) = 37.5\text{J}$ (91K)

$\Rightarrow -8\text{J}$, $V = 1.52\text{L}$

2.5 One mole of N_2 gas is contained at 273 K and a pressure of 1 atm. The addition of 3000 J of heat to the gas at constant pressure causes 832 J of work to be done during the expansion. Calculate

a. The final state of the gas
b. The values of ΔU and ΔH for the change of state
c. The values of c_v and c_p for N_2

Assume that nitrogen behaves as an ideal gas, and that the change of state is conducted reversibly.

$PV = nRT$

$V = \frac{0.082 \times 1 \times 376}{1}$

#a N_2, $T = 376K$, $P = 1atm$, $q = 3000J$, $W = -832J$
$V = 31L$

#b $\Delta U = q + w = 3000 - 832 = 2168J$

$P = 1atm$

$dU = C_v dT$

$\Delta U = \frac{5}{2} \times 8.314 J/K \times (T_2 - 273K) = 2168J$

$\Rightarrow T_2 = 376K$

$dH = C_p dT$

$\Delta H = \frac{7}{2} \times 8.314 J/K \times (\cancel{376K - 273K}) = 3020J$ (104K)

#C

$C_v = \frac{5}{2} \times 8.314 J/K \doteq 21 J/K$

$C_p = \frac{7}{2} \times 8.314 J/K \doteq 30 J/K$

3.1 The initial state of 1 mole of a monatomic ideal gas is P = 10 atm and T = 300 K. Calculate the change in the entropy of the gas for

a. An isothermal decrease in the pressure to 5 atm
b. A reversible adiabatic expansion to a pressure of 5 atm
c. A constant-volume decrease in the pressure to 5 atm

1 mol, $P = 10$ atm, $T = 300$K

#a $\Delta S = \frac{q}{T}$ $\Delta U = q + w$, $\Delta U = 0$

$\Rightarrow q = -w$

$$V_1 = \frac{1\text{mol} \times 0.082\,\text{L}\cdot\text{atm/K}\cdot\text{mol} \times 300\text{K}}{10\text{atm}} = 2.5\text{L}$$

$$V_2 = V_1 \times 2 = 5\text{L}$$

$$W = -\int_{2.5}^{5} \frac{nRT}{V}\,dV = -1\text{mol} \times 0.082\,\text{L}\cdot\text{atm/K}\cdot\text{mol} \times 300\text{K} \times \ln 2 = -17.05\,\text{L}\cdot\text{atm}$$

$$\Rightarrow -17.05\,\text{L}\cdot\text{atm} \times 101.325\,\text{J/L}\cdot\text{atm} = -1728\text{J}$$

$$q = -W = 1728\text{J}$$

$$\Delta S = \frac{q}{T} = \frac{1728\text{J}}{300\text{K}} = 5.76\,\text{J/K}$$

#b $q=0$, $\Delta S = \frac{q}{T} = 0\,J/K$

#C

const. Volume $\Rightarrow W=0$

$\Delta U = q+W$, $q=\Delta U$

$$V = \frac{nRT}{P_1} = 2.4L$$

$$T_2 = \frac{P_2 V}{nR} = \frac{5atm \times 2.4L}{1mol \times 0.082 L\cdot atm/K\cdot mol} = 152K$$

$$\Delta S = \int \frac{dq}{T} = n\int_{300}^{152} \frac{C_v}{T} dT$$

$$= 1mol \times \frac{3}{2} \times 8.314 J/K\cdot mol \times \ln\left(\frac{152}{300}\right)$$

$$= -8.47 J/K$$

3.2 One mole of a monatomic ideal gas is subjected to the following sequence of steps:

a. Starting at 300 K and 10 atm, the gas expands freely into a vacuum to triple its volume.

b. The gas is next heated reversibly to 400 K at constant volume.

c. The gas is reversibly expanded at constant temperature until its volume is again tripled.

d. The gas is finally reversibly cooled to 300 K at constant pressure.

Calculate the values of q and w and the changes in U, H, and S.

*a $T=300K,\ p=10\,atm,\ 1mol\ ,\ V \rightarrow 3V$

$$\Delta U=\Delta H\ ,\ q=w\ ,\ w=-\int \frac{nRT}{V}dV$$

$$=-1mol \times 8.314 J/K\cdot mol \times 300K \times \ln 3 = -2740J$$

$$\Delta S=\frac{2740J}{300K}=9.13J/K$$

*b $T=300K \rightarrow 400K$, const. Volume

$$w=0\ ,\ \Delta U=q+\cancel{w}^{0}\ ,\ \Delta U=\int C_V dT$$

$$=\frac{3}{2}\times 1mol \times 8.314 J/K\cdot mol \times (400K-300K)=1247J$$

$$\Delta H=\frac{5}{2}\times 1mol \times 8.314 J/K\cdot mol \times (400K-300K)=2078J$$

$$\Delta S=\int \frac{nC_V}{T}dT=1mol \times \frac{3}{2}\times \ln\left(\frac{400K}{300K}\right)\times 8.314J/K\cdot mol=3.59J/K$$

*c Isothermally $3V \rightarrow 9V$

$T=400K,$

$$\Delta U=0\ ,\ w=-\int p\,dV=-\int_{3V}^{9V}\frac{nRT}{V}dV$$

$= -1\,mol \times 8.314\,J/K\cdot mol \times 400\,K \times \ln(3) = -3653\,J$ //

$\Delta H = \cancel{\Delta U}^{\,0} + \Delta(n\cancel{RT})^{\,0} = 0\,J$ //

$\Delta S = \int \frac{dq}{T} = \frac{3653\,J}{400\,K} = 9.13\,J/K$ //

*d const. pressure, $T = 400\,K \rightarrow 300\,K$

$\Delta U = \int C_v dT = 1\,mol \times \frac{3}{2} \times 8.314\,J/K\cdot mol \times (300\,K - 400\,K)$

$= -1247\,J$ //

$\Delta H = \int C_p dT = 1\,mol \times \frac{5}{2} \times 8.314\,J/K\cdot mol \times (300\,K - 400\,K)$

$= -2079\,J$ //

$\Delta S = \int \frac{dq}{T} = \int \frac{nC_p}{T} dT$

$= 1\,mol \times \frac{5}{2} \times 8.314\,J/K\cdot mol \times \ln\left(\frac{4}{3}\right) = 5.98\,J/K$ //

Total $\Rightarrow$ $\Delta U = \Delta H = 0$. $q = w = 2823\,J$,

$\Delta S = 15.88\,J/K$ //

3.3 One mole of a monatomic ideal gas undergoes a reversible expansion at constant pressure, during which the entropy of the gas increases by 14.41 J/K and the gas absorbs 6236 J of thermal energy. Calculate the initial and final temperatures of the gas. One mole of a second monatomic ideal gas undergoes a reversible isothermal expansion, during which it doubles its volume, performs 1729 J of work, and increases its entropy by 5.763 J/K. Calculate the temperature at which the expansion was conducted.

const. pressure

$\Delta S = 1\,mol \times \frac{5}{2} \times 8.314\,J/K \cdot mol \times \ln\left(\frac{T_2}{T_1}\right) = 14.41\,J/K$

$\ln\left(\frac{T_2}{T_1}\right) = 0.69$, $\left(\frac{T_2}{T_1}\right) = 2$, $T_2 = 2T_1$

$q = \int C_p\,dT$, $1\,mol \times \frac{5}{2} \times 8.314\,J/K \cdot mol \times (T_1) = 6236\,J$

$T_1 = 300\,k$, $T_2 = 2T_1 = 600\,k$ //

$* \; V \rightarrow 2V$

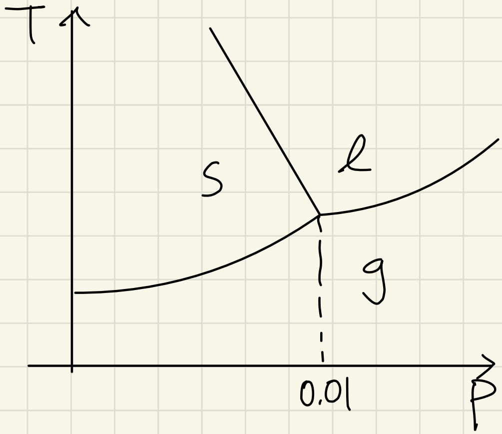
$W = -1729\,J$

$\Delta S = 5.763\,J/K$

Isothermal $\Rightarrow \Delta U = 0$, $q = -W = 1729\,J$ //

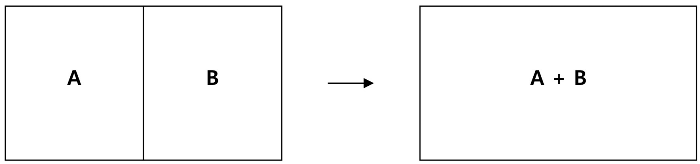
$\Delta S = \frac{1729\,J}{T} = 5.763\,J/K \Rightarrow T = 300\,k$ //

7. 늦가을 자동차를 운전하면 유리창에 김 서림이 문제가 된다. 자동차 유리창에 김이 서리는 이유를 H_2O의 PT diagram을 이용하여 과학적으로 설명하시오. 이를 제거하기 위해 냉난방 장치를 이용할 경우 창 쪽으로 더운 공기가 나오게 하는 것이 현명한 가, 아니면 에어컨 바람이 나오게 하는 것이 현명한 가? 근거를 대고 설명하시오.



자동차 유리창 안팎의 온도차이로 인해 김이 서리게 된다.
즉, 겨울철 자동차 유리창 밖의 온도는 매우 낮다.
그러므로 자동차 내에서 에어컨을 틀어 차 안의 온도를 유리창 밖의 온도와 최대한 비슷하게 만들어주어야 한다.
이는 P-T diagram을 통해 설명하자면, 자동차 안팎의 압력은 동일하다. 또한 온도 차이로 인해 따뜻한 공기가 이슬점 이하의 물체에 부딪히게 되면서 급격히 응결되는 것이다.
즉, 따뜻한 공기가 나오게 하면 응결되는 수증기가 더 많아지게 되므로 현명하지 못하다. 그래서 에어컨을 트는 것이 현명하다.

8. 그림 왼쪽과 같이 분리되어 있던 두 종류의 gas 입자들은 칸막이를 제거할 경우 서로 섞여 균일한 혼합체를 이룬다. 각 gas 입자들은 자신들이 섞여 있어야 할 운명이라는 것을 미리 알고 있었을까? (서로 섞여야 한다는 어떤 force 같은 것을 느끼게 되는 걸까?) 이 문제에 대한 견해를 밝히시오.



개인적인 의견으로는 기체 분자들이 서로 섞여야 한다는 어떠한 force 같은 것을 느끼지는 못할 것이라고 생각합니다.

기체분자들은 다른 상태일 때보다 빠르고 자유롭게 움직이는 특성을 가지고 있습니다.

즉, (A)와 (B)에 각각 갇혀있는 기체 분자들 모두 칸막이가 열리기 전부터 움직이고 있었을 것입니다.

그래서 칸막이가 열린 후에도, 단순히 기체 분자들이 움직이는 거리의 범위가 넓어지면서 섞이게 되는 것이라고 생각합니다. 이러한 이유로 기체 분자들이 서로 섞이는 것을 미리 알고 있지는 않았을 것입니다.