2.1 An monatomic ideal gas at 300 K has a volume of 15 liters at a pressure of 15 atm. Calculate

a. The final volume of the system
b. The work done by the system
c. The heat entering or leaving the system
d. The change in the internal energy
e. The change in the enthalpy when the gas undergoes
   i. A reversible isothermal expansion to a pressure of 10 atm
   ii. A reversible adiabatic expansion to a pressure of 10 atm

The constant-volume molar heat capacity of the gas, c_v, has the value 1.5 R.

$T_1 = 300k$, $V_1 = 15L$, $P_1 = 15atm$. by $PV = nRT$, $n = P_1V_1/RT_1 = 15 \times 15 / 0.08206 \times 300 = 9.14 mol$, $P_2 = 10atm$

a) i). $P_1V_1 = P_2V_2$, $V_2 = P_1V_1/P_2 = 15 \times 15/10 = 22.5L$, ii) $P_1V_1^{\gamma} = P_3V_3^{\gamma}$, $15 \times 15^{\frac{5}{3}} = 10 \times V_3^{\frac{5}{3}}$, $V_3 = 19.13L$

b) i) 등온 과정에서, $W = nRT\ln\frac{V_2}{V_1} = 9.14 \times 8.3144 \times 300 \times \ln\frac{22.5}{15} = 9244 J$

ii) adiabatic, $w = -\Delta U = -nC_v\Delta T = -nC_v(T_3 - T_1) = -9.14 \times \frac{3}{2} \times 8.3144 \times (255 - 300) = 5130J$ ($\because T_3 = \frac{P_3V_3}{nR} = \frac{10 \times 19.13}{9.14 \times 0.08206} = 255k$)

c) i) $q = w = 9244J$, ii) $q = 0$

d) i) $\Delta U = 0$ ($\because \Delta T = 0$), ii) $\Delta U = -W = -5130J$

e) i) $\Delta H = 0$ ($\because \Delta T = 0$), ii) $\Delta H = nC_p(T_3 - T_1) = 9.14 \times \frac{5}{2} \times 8.3144 \times (255 - 300) = -8549J$

2.3 The initial state of a quantity of monatomic ideal gas is P = 1 atm, V = 1 liter, and T = 373 K. The gas is isothermally expanded to a volume of 2 liters and is then cooled at constant pressure to the volume V. This volume is such that a reversible adiabatic compression to a pressure of 1 atm returns the system to its initial state. All of the changes of state are conducted reversibly. Calculate the value of V and the total work done on or by the gas.

by $PV = nRT$, $n = \frac{P_1V_1}{RT} = \frac{1 \cdot 1}{0.08206 \times 373} = 0.0327 mol$

등온 팽창 후, $P_2 = \frac{P_1 \cdot V_1}{V_2} = \frac{1 \cdot 1}{2} = \frac{1}{2} atm$

등온조건에서, $P_1V_1^{\gamma} = P_3V_3^{\gamma}$ ($\gamma = \frac{5}{3}$)

$1 \cdot 1^{\frac{5}{3}} = \frac{1}{2} \cdot V_3^{\frac{5}{3}}$ $\therefore V_3 = 1.52L$

$W_{1\to2}(\text{isothermal}) = nRT\ln\frac{V_2}{V_1} = 0.0327 \times 8.3144 \times 373 \times \ln 2 = 70.3J$

$W_{2\to3}(\text{constant pressure}) = P(V_3 - V_2) = \frac{1}{2}(1.52 - 2) L \cdot atm = -24.5J$ ($1L \cdot atm = 101.325J$)

$W_{3\to1}(\text{adiabatic}) = -\Delta U = -nC_v(T_1 - T_3) = -0.0327 \times \frac{3}{2} \times 8.3144 (373 - 282) = -37.1J$

$\therefore W_{total} = 70.3 - 24.5 - 37.1 = 8.7J$

2.5 One mole of N_2 gas is contained at 273 K and a pressure of 1 atm. The addition of 3000 J of heat to the gas at constant pressure causes 832 J of work to be done during the expansion. Calculate

a. The final state of the gas
b. The values of ΔU and ΔH for the change of state
c. The values of c_v and c_p for N_2

Assume that nitrogen behaves as an ideal gas, and that the change of state is conducted reversibly.

a) by $PV = nRT$, $V_1 = \frac{nRT_1}{P_1} = \frac{1 \times 0.08206 \times 273}{1} = 22.4L$

등온 과정 중에, $W = P(V_2 - V_1) = 1 \times (V_2 - 22.4) L \cdot atm = 832J$, $V_2 - 22.4 = 832/101.325$, $\therefore V_2 = 30.61L$

$T_2 = \frac{P_2V_2}{nR} = \frac{1 \times 30.61}{1 \times R} = \frac{30.61}{0.08206} = 373k$

b, c) $\Delta U = q - W = 3000 - 832 = 2168J$

$= 1 \times C_v(T_2 - T_1) = C_v(373 - 273) = 100 \cdot C_v$, $\therefore C_v = 21.7 J/k \cdot mol$

$q_p = \Delta H = 3000J = 1 \times C_p(373 - 273) = 100 \cdot C_p$, $\therefore C_p = 30 J/k \cdot mol$

3.1 The initial state of 1 mole of a monatomic ideal gas is $P = 10$ atm and $T = 300$ K. Calculate the change in the entropy of the gas for

a. An isothermal decrease in the pressure to 5 atm
b. A reversible adiabatic expansion to a pressure of 5 atm
c. A constant-volume decrease in the pressure to 5 atm

$P_1 = 10\,atm$, $T_1 = 300\,k$, by $PV = nRT$, $V_1 = nRT_1/P_1 = 1 \times 0.08206 \times 300/10 = 2.462\,L$

a) 등온 조건으로 $P_2 = 5\,atm$, $T_2 = 300\,k$

$V_2 = \frac{P_1V_1}{P_2} = \frac{10 \cdot 2.462}{5} = 4.924\,L$

$\Delta S = nR\ln\frac{V_2}{V_1} = 1 \times 8.3144 \times \ln 2 = 5.76\ J/k$

b) adiabatic 이므로 $q=0$, reversible 조건에서 $\Delta S = 0\ J/k$

c) constant volume 조건에서, $P_2 = 5\,atm$ 으로 $P_1V_1 = P_2V_2$ 에서 $V_2 = 2.462\,L$

$T_2 = \frac{P_2V_2}{nR} = \frac{5 \times 2.462}{1 \times 0.08206} = 150\,k$

$\Delta S = nC_v\ln\frac{T_2}{T_1} = 1 \times \frac{3}{2} \times 8.3144 \times \ln\frac{1}{2} = -8.65\ J/k$

3.2 One mole of a monatomic ideal gas is subjected to the following sequence of steps:

a. Starting at 300 K and 10 atm, the gas expands freely into a vacuum to triple its volume.
b. The gas is next heated reversibly to 400 K at constant volume.
c. The gas is reversibly expanded at constant temperature until its volume is again tripled.
d. The gas is finally reversibly cooled to 300 K at constant pressure.

Calculate the values of q and w and the changes in U, H, and S.

a) $T_1 = 300\,k$, $P_1 = 10\,atm$, by $PV = nRT$, $V_1 = 1 \times 0.08206 \times 300/10 = 2.462\,L$

1→2 (freely): $T_2 = T_1 = 300\,k$, $V_2 = 3 \times V_1 = 7.386\,L$, by $P_1V_1 = P_2V_2$, $P_2 = \frac{P_1}{3} = 3.33\,atm$

$\Delta U = q - W = 0$, $q = 0$, $W = 0$, $\Delta H = 0$, $\Delta S = nR\ln\frac{V_2}{V_1} = 1 \times 8.3144\ln 3 = 9.134\ J/K$

b) 2→3 (reversibly, 정적)

$T_3 = 400\,k$, $V_3 = V_2 = 7.386\,L$, by $\frac{T_3}{P_3} = \frac{T_2}{P_2}$, $P_3 = \frac{T_3}{T_2} \cdot P_2 = \frac{4}{3} \times 3.33 = 4.44\,atm$

$W = 0$ ($\because \Delta V = 0$), $\Delta U = q = C_v(T_3 - T_2) = \frac{3}{2} \cdot 8.3144 \times 100 = 1247\,J$

$\Delta H = C_p(T_3 - T_2) = \frac{5}{2} \times 8.3144 \times 100 = 2078\,J$ $\Delta S = nC_v \cdot \ln\frac{T_2}{T_1} = 1 \times \frac{3}{2} \times 8.3144 \times \ln\frac{400}{300} = 3.588\,J/K$

c) 3→4 (reversibly, 등온)

$T_4 = 400\,k$, $V_4 = 3V_3 = 22.158\,L$, by $P_3V_3 = P_4V_4$, $P_4 = \frac{P_3}{3} = 1.48\,atm$

$\Delta U = \Delta H = 0$, $q - W = 0$, $q = W = nRT_4\ln\frac{V_4}{V_3} = 1 \times 8.3144 \times 400 \times \ln 3 = 3654\,J$, $\Delta S = \frac{q}{T} = \frac{3654}{400} = 9.134\,J/K$

d) 4→5 (reversibly, 등압)

$T_5 = 300\,k$, $P_5 = P_4 = 1.48\,atm$, by $PV = nRT$, $V_5 = \frac{1 \times 0.08206 \times 300}{1.481} = 16.623\,L$

$\Delta U = nC_v(T_5 - T_4) = 1 \times \frac{3}{2} \times 8.3144(300 - 400) = -1247\,J = q_p - W$, $W = q_p - \Delta U = -2078 + 1247 = -831\,J$

$\Delta H = nC_p(T_5 - T_4) = 1 \times \frac{5}{2} \times 8.3144(300 - 400) = -2078\,J = q_p$

$\Delta S = nC_p\ln\frac{T_5}{T_4} = 1 \times \frac{5}{2}\ln\frac{300}{400} = -5.980\,J/K$

$\Delta U_{total} = 0$
$\Delta H_{total} = 0$
$W = 2823\,J$
$q = 2823\,J$
$\Delta S = 15.88\,J/K$

3.3 One mole of a monatomic ideal gas undergoes a reversible expansion at constant pressure, during which the entropy of the gas increases by 14.41 J/K and the gas absorbs 6236 J of thermal energy. Calculate the initial and final temperatures of the gas. One mole of a second monatomic ideal gas undergoes a reversible isothermal expansion, during which it doubles its volume, performs 1729 J of work, and increases its entropy by 5.763 J/K. Calculate the temperature at which the expansion was conducted.

i) constant pressure

$q = 6236\,J = nC_p(T_2 - T_1) = 1 \times \frac{5}{2}(T_2 - T_1)$, $T_2 - T_1 = 300$ ⋯ ①

$\Delta S = 14.41\,J/K = nC_p\ln\frac{T_2}{T_1} = 1 \times \frac{5}{2} \times 8.3144 \times \ln\frac{T_2}{T_1}$, $\frac{T_2}{T_1} = 2$ ⋯ ②

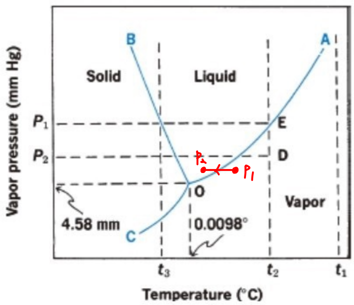
by ①, ② $T_2 = 600\,k$, $T_1 = 300\,k$

ii) isothermal

$\Delta U = q - W = 0$, $q = W = 1729\,J$

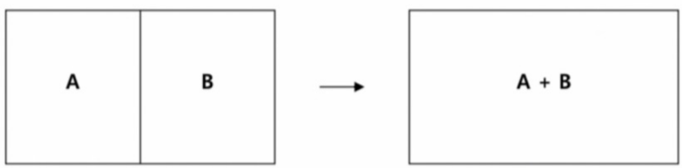
$\Delta S = \frac{q}{T} = 5.763\,J/K$, $T = \frac{1729}{5.763}\,k = 300\,k$

7. 늦가을 자동차를 운전하면 유리창에 김 서림이 문제가 된다. 자동차 유리창에 김이 서리는 이유를 H_2O의 PT diagram을 이용하여 과학적으로 설명하시오. 이를 제거하기 위해 냉난방 장치를 이용할 경우 창 쪽으로 더운 공기가 나오게 하는 것이 현명한 가, 아니면 에어컨 바람이 나오게 하는 것이 현명한 가? 근거를 대고 설명하시오.



저온철, 창문을 기준으로 밖과 안의 온도 차이가 나게 되어 차 내부의 수증기가 차가운 창문을 만나서 김이 서리게 된다. 이를 해결하기 위하여 에어컨을 틀음으로서 실내 온도를 낮추고 제습효과로 습도를 낮추어 김 서림 방지 효과를 얻을 수 있다.

8. 그림 왼쪽과 같이 분리되어 있던 두 종류의 gas 입자들은 칸막이를 제거할 경우 서로 섞여 균일한 혼합체를 이룬다. 각 gas 입자들은 자신들이 섞여 있어야 할 운명이라는 것을 미리 알고 있었을까? (서로 섞여야 한다는 어떤 force 같은 것을 느끼게 되는 걸까?) 이 문제에 대한 견해를 밝히시오.



엔트로피 증가 요건 중 하나는 기체가 위치할 수 있는 부피의 증가이다. 따라서 칸막이를 제거함으로서, 부피가 증가하여 기체가 균일하게 퍼져 위치할 수 있는 경우의 수가 늘어나서 확률이 증가 한다. 즉, 기체는 가장 안정한 상태로 가기 위해 자발적인 반응으로 엔트로피가 증가하는 퍼져나가는 운동을 한다. 이것이 기체 분자들이 서로 섞이려 하는 force에 따라 기체가 움직이는 것으로 보인다.