1. (a)

$dU = TdS - PdV$, $dS = \frac{dU}{T} + \frac{P}{T}dV = \frac{C_v}{T}dT + \frac{R}{V}dV$

Isothermal decrease → $dT = 0$

$\therefore \Delta S = \int_{V_1}^{V_2} \frac{R}{V}dV = R\ln\frac{V_2}{V_1} = R\ln\frac{P_1}{P_2} = R\ln 2$

$= 5.76 J/K$

(b) reversible adiabatic → $q = 0$

$dS = \frac{dq}{T}$, $\Delta S = 0$

(c) $dV = 0$

$\therefore dU = \frac{C_v}{T}dT$

$\therefore \Delta S = \int_{T_1}^{T_2} \frac{C_v}{T}dT = C_v\ln\left(\frac{T_2}{T_1}\right) = C_v\ln\left(\frac{P_2}{P_1}\right) = C_v\ln\frac{1}{2}$

$= -\frac{3}{2}R\ln 2 = -8.64 J/K$

2. $P_0 = 10 atm$, $T_0 = 300K$

$V_0 = \frac{RT_0}{P_0} = 2.46 L$

$V_a = 3V_0 = 7.38L$, $P_a = \frac{1}{3}P_0 = 3.33 atm$

$q_a = 0$, $W_a = 0$, $\Delta U_a = 0$, $\Delta H_a = 0$, $\Delta S_a = R\ln\frac{V_a}{V_0} = 9.13 J/K$

$V_b = V_a = 7.38L$. $T_b = 400K$. $P_b = \frac{4}{3}P_a = 4.44 atm$

$W_b = 0$, $\Delta U_b = C_v\Delta T = \frac{3}{2}R\Delta T = 1247.1 J$

$\Delta H_b = \Delta U_b + \Delta(PV) = \Delta U_b + V\Delta P = C_p\Delta T = 2078.5 J$

$dU = TdS - PdV$, $dS = \frac{dU}{T}$, $\Delta S_b = C_v\ln\frac{T_b}{T_a} = 3.59 J/K$

$q_b = \Delta U_b = 1247.1 J$

$T_c = T_b$, $V_c = 3V_b = 22.14 L$. $P_c = \frac{1}{3}P_b = 1.48 atm$

$W_c = q_c = \int PdV = RT\ln\frac{V_c}{V_b} = 3.61 kJ$

$\Delta U_c = 0$, $\Delta H_c = 0$ ($\because \Delta T = 0$)

$\Delta S_c = R\ln\frac{V_c}{V_b} = 9.13 J/K$

$P_d = P_c = 1.48 atm$, $T_d = 300K$, $V_d = 16.6 L$

$W_d = \int PdV = P\Delta V = -0.831 kJ$ $q_d = W_d + \Delta U_d = -2078.5J = \Delta H_d$

$\Delta U_d = C_v\Delta T = -1247.1 J$ $\Delta S_d = C_p\ln\frac{T_d}{T_c} = -5.98 J/K$

$\therefore W = 2.78 kJ$

$q = 2.78 kJ$

$\Delta U = 0$

$\Delta H = 0$

$\Delta S = 15.37 J/K$

3. (a) $\frac{\partial z}{\partial x} = 2(x-2)$. $\frac{\partial z}{\partial y} = 2(y-2)$

$\rightarrow (2,2)$: critical point

$f_{xx}=2$, $f_{yy}=2$, $f_{xy}=0$, $D(2,2)=4>0$

$\therefore$ (2,2)에서 extreme value 가짐.

$\therefore$ 4

(b) $y=1-x$

$z=(x-2)^2+(-1-x)^2+4$

$= 2x^2-2x+9 = 2(x-\frac{1}{2})^2+\frac{17}{2}$

$\therefore$ $x=\frac{1}{2}$, $y=\frac{1}{2}$ 일 때 최소, 이 때 $z=\frac{17}{2}$

(c) $z=f(x,y)$

$g(x,y)=x+y-1$

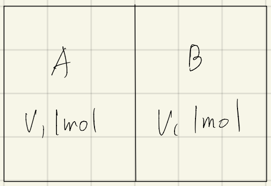
$\nabla f = \lambda \nabla g$

$\rightarrow \frac{\partial f}{\partial x}=\lambda$, $\frac{\partial f}{\partial y}=\lambda$, $\lambda = 2(x-2)=2(y-2)$

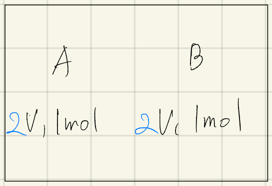
$\rightarrow x=y$, $x+y=1$이므로 $x=y=\frac{1}{2}$ 일 때 최소,

이 때 $z=\frac{17}{2}$

4.



(a)



$\Delta S_A = nR\ln\frac{V_f}{V_i} = R\ln 2$

$\Delta S_B = R\ln 2$

$\Delta S = 2R\ln 2$

(b) $\Delta S_A = 2R\ln 2$

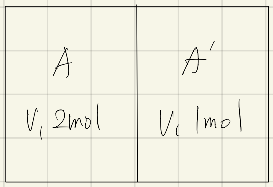
$\Delta S_B = R\ln 2$

$\Delta S = 3R\ln 2$

(c) (a) 경우

$\rightarrow \Delta S = 0$ (no condition changed)

(b) 경우



↓

2V, 3mol

$\Delta S_A = 2R\ln\frac{\frac{2}{3}V}{\frac{1}{2}V} = 2R\ln\frac{4}{3}$

$\Delta S_B = R\ln\frac{\frac{2}{3}V}{V} = R\ln\frac{2}{3}$

$\therefore \Delta S = R\ln\frac{32}{27}$