Problem set #2 2019 0014 김동규

1. The initial state of one mole of a monatomic ideal gas is $P = 10$ atm and $T = 300$ K. Calculate the change in the entropy of gas

$(V = \frac{RT}{P} = \frac{0.082 \times 300}{10} = 2.46$ L

(a) isothermal decrease in the pressure to 5 atm

$P_2 = 5 \quad T_2 = 300 \quad V_2 = \frac{RT_2}{P_2} = \frac{0.082 \times 300}{5} = 4.92$ L

$$\Delta S = nR \ln \frac{V_2}{V_1} + \cancel{nC_v \ln \frac{T_2}{T_1}}$$

$$= 8.3144 \times \ln \frac{4.92}{2.46} = 5.76 \text{ J/K}$$

(b) a reversible adiabatic expansion to a pressure of 5 atm

가역 단열 팽창에서 $q = 0$ 이므로 $\Delta S = \frac{q}{T} = 0$

(c) a constant-volume, decrease in the pressure to 5 atm.

등적과정이므로 $\Delta S = \cancel{nR \ln \frac{V_2}{V_1}} + nC_v \ln \frac{T_2}{T_1}$

$$= \frac{3}{2} \times 8.3144 \times \ln \frac{\frac{5 \times 2.46}{0.082}}{300} = -8.64 \text{ J/K}$$

1. Start at 300K, 10 atm, 2.462 L

(a)

1→2 gas expands freely into a vacuum to triple its volume

이상기체이므로 자유팽창 시 $\Delta T = 0$, $T_2 = 300K$

$dU = \delta q - \delta w = 0$, $q = 0$이므로 $W = 0$

$H = U + PV$ 에서 $\Delta H = 0 + nR\cancel{\Delta T} = 0$

$\Delta S = nR \ln \frac{3V}{V} + \cancel{nC_v \ln \frac{T_2}{T_1}} = 8.3144 \ln 3 = 9.134\ J/K$

(b) gas heated reversibly to 300 → 400K, at constant volume

$q = \Delta U = \frac{3}{2} \times R \times 100 = 1.5 \times 8.3144 \times 100 = 1247\ J$, $w = 0$

$\Delta H = \frac{5}{2} R(\Delta T) = \frac{5}{2} \times 8.3144 \times 100 = \frac{5}{3} \times 1247 = 2078\ J/K$

$\Delta S = C_v \ln \frac{400}{300} = 1.5 \times 8.3144 \times \ln \frac{4}{3} = 3.58\ J/K$

(c) The gas is reversibly expanded at constant T until V → 3V

$\Delta T = 0$ 이므로 $\Delta U = 0$, $\Delta H = 0$

$dU = \delta q - \delta w = 0$, $q = w = nRT \ln 3 = 8.3144 \times 400 \ln 3$

$= 3654\ J$

$\Delta S = \frac{q}{T} = \frac{3654}{400} = 9.13\ J/K$

(d) Reversibly cooled to 300K at constant pressure.

$\Delta U = \frac{3}{2} R \Delta T = \frac{3}{2} \times 8.3144 \times (-100) = -1247\ J$

$\Delta H = \Delta U + P\Delta V = nC_p \Delta T = \frac{5}{2} \times 8.3144 \times (-100) = -2078\ J$

$w = -2078 + 1247 = -831\ J$, $\Delta S = nC_p \ln \frac{V'}{V} = \frac{5}{2} \times 8.3144 \times \ln \frac{3}{4} = -5.98\ J/K$

3.

(a) Find the extreme value of the function
$z = (x-2)^2 + (y-2)^2 + 4$

$f(x, y) = z = (x-2)^2 + (y-2)^2 + 4$

$f_x = \frac{\partial z}{\partial x} = 2x - 4$, $f_{xx} = 2$
$f_y = \frac{\partial z}{\partial y} = 2y - 4$, $f_{yy} = 2$, $f_{xy} = 0$

모든 (x, y)에서 두 편도함수 f_x, f_y가 존재하므로 임계점은
$f_x = f_y = 0$ 을 만족할 때이다. 이때 $x = 2$, $y = 2$ 이다

$(2,2)$ 에서 판별식 $D(2,2) = f_{xx}(2,2) f_{yy}(2,2) - \{f_{xy}(2,2)\}^2$ 이므로

$D(2,2) = 2 \times 2 - 0^2 = 4 > 0$ 이다.

$D(2,2) > 0$ 이고 $f_{xx}(2,2) = 2 > 0$ 이므로 $f(x,y)$는 $(2,2)$에서
극솟값을 가진다.

(b) $y = 1 - x$ 이므로 y를 소거해보자. $z = (x-2)^2 + (1-x-2)^2 + 4$

$z = x^2 - 4x + 4 + 1 + 2x + x^2 + 4 = 2x^2 - 2x + 9$

$\frac{dz}{dx} = 4x - 2 = 0$, $x = \frac{1}{2}$

$\frac{d^2z}{dx^2} = 4 > 0$, Minimum value is $2 \cdot \frac{1}{4} - 2 \cdot \frac{1}{2} + 9 = \frac{17}{2}$ at $(\frac{1}{2}, \frac{1}{2})$

(c) $f(x, y) = z = (x-2)^2 + (y-2)^2 + 4$, $g(x, y) = x + y - 1$ 이라고 하자
$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = (2x-4, 2y-4)$, $\nabla g = \left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}\right) = (1, 1)$ 이다.

$\nabla f = \lambda \nabla g$, $a + b = 1$ 을 만족하므로,

$$\begin{cases} 2a - 4 = \lambda \\ 2b - 4 = \lambda \\ a + b = 1 \end{cases} \Rightarrow \begin{cases} a - b = 0 \\ a + b = 1 \end{cases} \Rightarrow a = \frac{1}{2}, \ b = \frac{1}{2}, \ \text{min value} = f(\tfrac{1}{2}, \tfrac{1}{2}) = \frac{17}{2}$$

4.

(a) partition removed, A 와 B 모두 부피가 2배 증가한다.

온도는 일정 (이상기체) 하므로 등온 과정이다.

$\Delta S = nR \ln \frac{V_2}{V_1}$ 에서, $\Delta S_A = R \ln 2$, $\Delta S_B = R \ln 2$

$\therefore \Delta S_{total} = \Delta S_A + \Delta S_B = 2R \ln 2$

(b) 만약 A가 2mole 이었다면

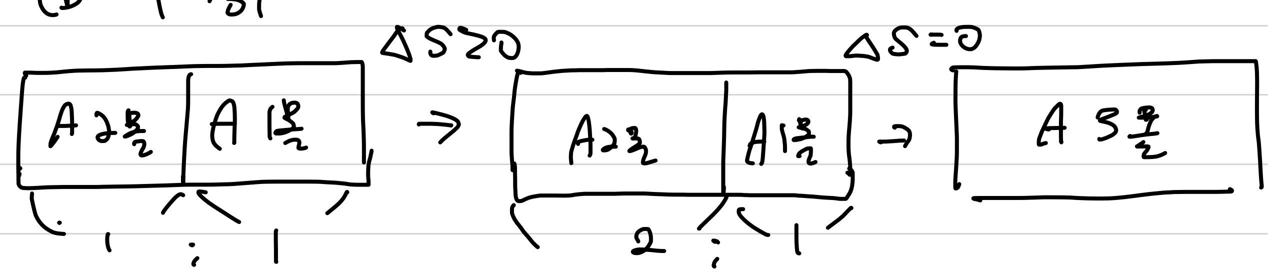
$\Delta S_A = 2R \ln 2$, $\Delta S_B = R \ln 2$

$\therefore \Delta S_{total} = \Delta S_A + \Delta S_B = 3R \ln 2$

(c) 양쪽에 모두 기체 A가 들어 있었다면

i) (a)의 경우 칸막이를 제거해도 차이가 없으므로 $\Delta S = 0$

ii) (b)의 경우



왼쪽: $\frac{1}{2}V \rightarrow \frac{2}{3}V$ 로 부피 증가, $\Delta S = 2R \ln \frac{4}{3}$

오른쪽: $\frac{1}{2}V \rightarrow \frac{1}{3}V$ 로 부피 감소, $\Delta S = R \ln \frac{\frac{1}{3}}{\frac{1}{2}} = R \ln \frac{2}{3}$

$\therefore \Delta S_{total} = 2R \ln \frac{4}{3} + R \ln \frac{2}{3} = R \ln\left(\frac{16}{9} \times \frac{2}{3}\right)$

$= R \ln \frac{32}{27}$