

1. $P_1 = 10 \text{ atm}$, $T_1 = 300 \text{ K}$ (a) $P_2 = 5 \text{ atm}$, $T_1 = T_2 = 300 \text{ K}$, $V_2 = 2V_1 = 4.92 \text{ L}$, $V_1 = 2.46 \text{ L}$
 $\Delta U = q + w = 0$, $q = -w = nRT = \int_{V_1}^{V_2} \frac{1}{V} dV = 1728.85 \text{ J}$, $\Delta S = \int_{T_1}^{T_2} \frac{dq}{T} = \frac{1}{T} \int_{V_1}^{V_2} \frac{1}{V} dV = \frac{1}{T} \ln \frac{V_2}{V_1} = \frac{1}{300} \ln 2 = 5.76 \text{ J/K}$

(b) In adiabatic process, $dq = 0$, $\Delta S = \int \frac{dq}{T} = 0$

(c) $w = 0$, $dq = dU = nC_V dT$, $\Delta S = \int \frac{dq}{T} = nC_V \int_{T_1}^{T_2} \frac{1}{T} dT = -8.64 \text{ J/K}$, $T_2 = \frac{P_2 V_2}{nR} = \frac{1}{2} T_1$

2. $T_1 = 300 \text{ K}$, $P_1 = 10 \text{ atm}$, $V = \frac{nRT}{P} = 2.46 \text{ L}$

(a) $T_1 = 300 \text{ K}$, $T_2 = 100 \text{ K}$, $T_3 = 300 \text{ K}$, $T_4 = 200 \text{ K}$, $T_1 = T_2$, $\Delta U = 0$, $\Delta H = 0$, $w = PdV = 0$ ($P = 0$), $q = -w = 0$

$\Delta S = Nk \ln 2 = 1 N_A \times \frac{R}{N_A} \ln 2 = 9.134 \text{ J/K}$, $V = 2.46 \text{ L}$

(b) $T_1 = 400 \text{ K}$, $V_1 = 3V_2 = 1.38 \text{ L}$, $P_1 = \frac{nRT_1}{V_1} = \frac{40}{2.7} \text{ atm}$

$w = \int P dV = 0$, $q = \Delta U = nC_V \Delta T = 1240.1 \text{ J}$, $\Delta H = nC_P \Delta T = 2018.5 \text{ J}$, $\Delta S = \int_{T_1}^{T_2} nC_V \frac{1}{T} dT = 3.588 \text{ J/K}$

(c) $T_1 = 300 \text{ K}$, $V_1 = 3V_2 = 22.14 \text{ L}$, $P_1 = \frac{nRT_1}{V_1} = \frac{40}{2.7} \text{ atm}$

$\Delta U = 0$, $\Delta H = 0$, $q = \int P dV = -nRT \int_{V_1}^{V_2} \frac{1}{V} dV = -3653.54 \text{ J}$, $q = -w = 3653.54 \text{ J}$, $\Delta S = \int \frac{dq}{T} = \frac{1}{T} \int dT = \frac{99}{T} = 9.134 \text{ J/K}$

(d) $T_1 = 300 \text{ K}$, $P_1 = P_2 = \frac{40}{2.7} \text{ atm}$, $V_1 = \frac{nRT_1}{P_1} = 16.605 \text{ L}$

$\Delta U = nC_V \Delta T = -1240.1 \text{ J}$, $q = \Delta H = nC_P \Delta T = -2018.5 \text{ J}$, $w = \Delta U = q = -3653.54 \text{ J}$, $\Delta S = \int \frac{dq}{T} = nC_P \int_{T_1}^{T_2} \frac{1}{T} dT = -9.979 \text{ J/K}$

3. $Z' = 0$, $1 \leq x \leq 2$, $0 \leq y \leq 1$, $f(x, y) = 2x - 4$, $(f_x(1, 1), f_y(1, 1)) = (0, 0)$, $x = 2$, $y = 2$

$x + y = 1$, $y = 1 - x$, $2 = (x-1)^2 + (-x-1)^2 + 4$, $2(x-1)^2 + 2x^2 = 2x^2 - 4x + 2 + 2x^2 = 4x^2 - 4x + 2$, $x = 0.5$, $y = 0.5$

$(H, C) F(x, y, \lambda) = (x-1)^2 + (y-1)^2 + \lambda(x+y-1) + 4$, $F_x = 2(x-1) + \lambda = 0$, $x = \frac{\lambda}{2} + 1$, $y = \frac{\lambda}{2} + 1$

$F_y = 2(y-1) + \lambda = 0$, $-\lambda + 4 = 1$, $\lambda = 3$, $(x, y) = (\frac{5}{2}, \frac{5}{2})$

$F_z = x + y - 1 = 0$

$\Delta S = \Delta S_A + \Delta S_B =$

(a) $\Delta S_A = Nk \ln 2 = 1 N_A \frac{R}{N_A} \ln 2 = 9.134 \text{ J/K}$

$\Delta S_B = Nk \ln 2 = 1 N_A \frac{R}{N_A} \ln 2 = 9.134 \text{ J/K}$

(b) $\Delta S_A = Nk \ln 2 = 2 N_A \frac{R}{N_A} \ln 2 = 18.268 \text{ J/K}$

$\Delta S_B = Nk \ln 2 = 1 N_A \frac{R}{N_A} \ln 2 = 9.134 \text{ J/K}$

(c) $\Delta S = \Delta S_A + \Delta S_B = 18.268 \text{ J/K}$