1. (a) $\Delta S = \frac{Q}{T}$, $Q = q_{rev} = W_{max} = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{RT}{V} dV = RT\ln\left(\frac{V_2}{V_1}\right)$

Isothermal이므로 PV is constant. $P_1V_1 = P_2V_2$, $10atm \cdot V_1 = 5atm \cdot V_2$, $2V_1 = V_2$

$Q = RT\ln\left(\frac{V_2}{V_1}\right) = RT\ln 2$,

$\Delta S = \frac{Q}{T} = \frac{RT\ln 2}{T} = R\ln 2 = 5.763 J/K$

(b) $\Delta S = \frac{Q}{T}$, adiabatic process이므로 $Q=0$, 따라서 $\Delta S = 0$.

(c) $\Delta S = \frac{Q}{T}$, $q = dU + w = dU + PdV$

Constant-Volume 이므로 $dV = 0$, $q = dU$

$dU = nC_v dT = Q$, $dS = \frac{C_v dT}{T} = \frac{3}{2}R\frac{dT}{T}$

$\Delta S = \int_{S_1}^{S_2} dS = \int_{T_1}^{T_2} \frac{3}{2}R\frac{dT}{T} = \frac{3}{2}R\ln\left(\frac{T_2}{T_1}\right)$

$\frac{PV}{T}$ is constant, $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$, $\frac{T_2}{T_1} = \frac{P_2V_2}{P_1V_1} = \frac{1}{2}$

$\Delta S = \frac{3}{2}R\ln\left(\frac{T_2}{T_1}\right) = -\frac{3}{2}R\ln 2 = -8.644 J/K$

2. a. Vaccum으로 freely expands 하는 것은 이상기체에서 isothermal하다.

$\Delta U = nC_v\Delta T = 0$, $q = w$이다.

$w = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{RT}{V}dV = RT\ln\left(\frac{V_2}{V_1}\right) = RT\ln 3$

$w = RT\ln 3 = (8.314J/K)\cdot(300K)\cdot\ln 3 = 2.740kJ$

$q = w = 2.740kJ$, $\Delta U = 0$, $\Delta H = nC_p\underbrace{\Delta T}_{0} = 0$, $\Delta S = \frac{Q}{T} = \frac{RT\ln 3}{T} = R\ln 3 = 9.134J/K$

b. $\Delta U = nC_v\Delta T = C_v\Delta T = \frac{3}{2}R\cdot(100K) = 1.247kJ$

$q = \Delta U + w$, $w = \int \underbrace{PdV}_{0} = 0$, $q = \Delta U = 1.247kJ$

$\Delta H = C_p\Delta T = \frac{5}{2}R(100K) = 2.079kJ$,

$dU = TdS - PdV$, $dS = \frac{dU}{T} + \frac{\overset{\to 0}{PdV}}{T} = \frac{C_v dT}{T}$, $\Delta S = \int_{S_1}^{S_2} dS = \int_{T_1}^{T_2} \frac{C_v}{T}dT = C_v\ln\left(\frac{T_2}{T_1}\right) = \frac{3}{2}R\cdot\ln\left(\frac{4}{3}\right) = 3.588J/K$

C. $\Delta U = \Delta H = 0$ in constant temperature.

$\Delta U = q - w$, $q = w$.

$w = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{RT}{V}dV = RT\ln\left(\frac{V_2}{V_1}\right) = RT\ln 3 = (8.314J/K)\cdot(400K)\cdot\ln 3 = 3.654kJ$

$q = w = 3.654kJ$

$dS = \frac{\overset{\to 0}{dU}}{T} + \frac{PdV}{T} = \frac{PdV}{T}$, $\Delta S = \int_{S_1}^{S_2} dS = \int_{V_1}^{V_2} \frac{PdV}{T} = \frac{w}{T} = \frac{3.654kJ}{400K} = 9.135J/K$

d. $\Delta U = C_v\Delta T = \frac{3}{2}R(-100K) = -1.247kJ$

$w = \int_{V_1}^{V_2} PdV \overset{\text{P is constant}}{=} P\Delta V$

C 과정 이후 state들이 아래첨자 C, initial state에 아래 첨자 i로 표시할 때,

$\frac{P_iV_i}{T_i} = \frac{P_cV_c}{T_c}$, $P_c = \frac{P_iV_iT_c}{V_cT_i} = \frac{(10atm)V_i(400K)}{9V_i(300K)} = 1.48atm$

d 과정 이후 state를 아래첨자 d로 표시하면,

$\frac{P_cV_c}{T_c} = \frac{P_dV_d}{T_d}$, $\frac{V_d}{V_c} = \frac{T_d}{T_c} = \frac{3}{4}$, $V_d = \frac{3}{4}V_c$, $-\Delta V = \frac{1}{4}V_c = \frac{9}{4}V_i = \frac{9}{4}\cdot\frac{RT_i}{P_i} = \frac{9}{4}\cdot\frac{(0.0821L\cdot atm/K)}{10atm}\cdot 300K = 5.535L$

$w = P\Delta V = -(1.48atm)(5.535L) = -8.19atm\cdot L = -829.9J$

$q = \Delta U + w = -1.247kJ - 829.9J = -2.08kJ$

$\Delta H = C_p\Delta T = \frac{5}{2}R\cdot(-100K) = -2.079kJ$

$dS = \frac{dU}{T} + \frac{PdV}{T} \overset{\text{P is constant, } d(PV) = PdV}{=} \frac{dH}{T} = C_p\frac{dT}{T}$

$\Delta S = \int_{S_1}^{S_2} dS = \int_{T_1}^{T_2} C_p\frac{dT}{T} = C_p\ln\left(\frac{T_2}{T_1}\right) = \frac{5}{2}R\ln\left(\frac{3}{4}\right) = -5.979J/K$

3. (a) $z = f(x,y) = x^2 + y^2 - 4x - 4y + 12 = 0$

$\nabla f = \left(\frac{\partial x}{\partial f}, \frac{\partial y}{\partial f}\right)$

$\frac{\partial x}{\partial f} = 2x - 4$, $\frac{\partial y}{\partial f} = 2y - 4$. $\nabla f = (2x-4, 2y-4)$

$(2,2)$에서 $\nabla f = 0$, $(2,2)$에서 z는 extreme value를 가짐.

(b) $y = 1-x$일 때, $z = x^2 + (1-x)^2 - 4x - 4(1-x) + 12 = 2x^2 - 2x + 9$

$\frac{dz}{dx} = 4x - 2$, $x = \frac{1}{2}$, $y = \frac{1}{2}$에서 constraint minimum. $f\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{17}{2}$

(c) $g(x,y) = x + y - 1$이라고 할 때, $L = f + \mu g$

$\nabla L = \left(\frac{\partial L}{\partial x}, \frac{\partial L}{\partial y}\right) = (2x - 4 + \mu, 2y - 4 + \mu) = (0,0)$

$2x - 4 + \mu = 2y - 4 + \mu = 0$, $2x + 2y - 8 + 2\mu = 0$

Constraint에 의해 $x + y = 1$이므로 $2(x+y) - 8 + 2\mu = 2\mu - 6 = 0$, $\mu = 3$

$\nabla L = (2x - 1, 2y - 1) = (0,0)$

$x = \frac{1}{2}$, $y = \frac{1}{2}$, $f\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{17}{2}$ (minimum)

4. $\Delta U = 0$, $TdS = PdV$, $dS = \frac{P}{T} dV = \frac{nR}{V} dV$

$\Delta S = \int_{S_1}^{S_2} dS = \int_{V_1}^{V_2} \frac{nR}{V} dV = nR \ln\left(\frac{V_2}{V_1}\right)$

(a) $\Delta S = \Delta S_A + \Delta S_B$

$\Delta S_A = 1 \cdot R \cdot \ln 2 = R \ln 2$, $\Delta S_B = 1 \cdot R \cdot \ln 2 = R \ln 2$

$\Delta S = \Delta S_A + \Delta S_B = 2R \ln 2$

(b) $\Delta S = \Delta S_A + \Delta S_B$, $\Delta S_A = 2 \cdot R \cdot \ln 2 = 2R \ln 2$, $\Delta S_B = 1 \cdot R \cdot \ln 2$

$\Delta S = \Delta S_A + \Delta S_B = 3R \ln 2$

(c) - (a) $\Delta S = 0$ → 이미 uniform하게 분포됨.

(c) - (b) $\Delta S = \Delta S_L + \Delta S_R$

$\Delta S_L = 2 \cdot R \cdot \ln\left(\frac{4}{3}\right)$, $\Delta S_R = 1 \cdot R \cdot \ln\left(\frac{2}{3}\right)$

$\Delta S = \Delta S_L + \Delta S_R = R \ln\left(\frac{32}{27}\right)$