Department of Materials Science and Engineering
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AMSE205 Thermodynamics I

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Problem Set #2

1. The initial state of one mole of a monatomic ideal gas is P = 10 atm and T = 300 K. Calculate the change in the entropy of the gas for (a) an isothermal decrease in the pressure to 5 atm, (b) a reversible adiabatic expansion to a pressure of 5 atm, (c) a constant-volume decrease in the pressure to 5 atm.

2. One mole of monatomic ideal gas is subjected to the following sequence of steps:
   a. Starting at 300 K and 10 atm, the gas expands freely into a vacuum to triple its volume.
   b. The gas is next heated reversibly to 400 K at constant volume.
   c. The gas is reversibly expanded at constant temperature until its volume is again tripled.
   d. The gas is finally reversibly cooled to 300 K at constant pressure.

   Calculate the values of q and w and the changes in U, H and S.

3. (a) Find the extreme value of the function,

$$z = (x - 2)^2 + (y - 2)^2 + 4.$$

Find the constrained maximum of this function corresponding to the condition

$$x + y = 1$$

(b) by eliminating one variable and (c) by using a Lagrange undetermined multiplier method.

4. A rigid container is divided into two compartments of equal volume by a partition. One compartment contains 1 mole of ideal gas A at 1 atm, and the other compartment contains 1 mole of ideal gas B at 1 atm.
   (a) Calculate the entropy increase in the container if the partition between the two compartments is removed.
   (b) If the first compartment had contained 2 moles of ideal gas A, what would have been the entropy increase due to gas mixing when the partition was removed?
   (c) Calculate the corresponding entropy changes in each of the above two situations if both compartments had contained ideal gas A.

Thermodynamics I Problem Set #2. (Oct.12.2021)

1. The initial state of one mole of a monatomic ideal gas is P = 10 atm and T = 300 K. Calculate the change in the entropy of the gas for (a) an isothermal decrease in the pressure to 5 atm, (b) a reversible adiabatic expansion to a pressure of 5 atm, (c) a constant-volume decrease in the pressure to 5 atm.

$P_1 = 10 \quad n_1 = 1 \qquad \longrightarrow \qquad P_2 = 5 \quad n_2 = 1$

$T_1 = 300 \quad V_1 = 2.46\ell \qquad \qquad T_2 = 300 \quad V_2 = 4.92$

a) isothermal $\rightarrow \Delta U = 0, \ q = w$

$$w = \int P\,dV = \int_{V_1}^{V_2} \frac{nRT}{V}\,dV = nRT\ln\frac{V_2}{V_1} \rightarrow q = -w = -nRT\ln\frac{V_2}{V_1}$$

$$\Delta S = \frac{q}{T} = nR\ln\frac{V_2}{V_1} = 8.314 \times \ln\frac{4.92}{2.46} = 8.314\ln 2 = \boxed{5.763\ (J/K)}$$

b) reversible adiabatic expansion

$$P_1V_1^{\frac{5}{3}} = P_2V_2^{\frac{5}{3}} \rightarrow V_2 = V_1\cdot\left(\frac{P_1}{P_2}\right)^{\frac{3}{5}} = 3.73(\ell)$$

* adiabatic process 에서 $\boxed{\Delta S = 0}$

c) a constant-volume decrease in the pressure to 5 atm

$$dU = nc_v dT = \delta q - \delta w \rightarrow \delta q = nc_v dT + \delta w^{\approx 0} = nc_v dT$$

$$\int\frac{\delta q}{T} = \int dS = \int nc_v\cdot\frac{1}{T}dT \quad \frac{3}{2}\times 8.314\ln\frac{T_2}{T_1} = \frac{3}{2}\times 8.314\ln\frac{1}{2}$$

$$= \boxed{-8.64\ (J/K)}$$

$P_1 = 10 \quad n_1 = 1 \qquad \longrightarrow \qquad P_2 = 5 \quad n_2 = 1$

$T_1 = 300 \quad V_1 = 2.46 \qquad \qquad T_2 = 150 \quad V_2 = 2.46$

2. 2. One mole of monatomic ideal gas is subjected to the following sequence of steps: 3번.
   a. Starting at 300 K and 10 atm, the gas expands freely into a vacuum to triple its volume.
   b. The gas is next heated reversibly to 400 K at constant volume.
   c. The gas is reversibly expanded at constant temperature until its volume is again tripled.
   d. The gas is finally reversibly cooled to 300 K at constant pressure.

   Calculate the values of q and w and the changes in U, H and S.

a) 300K, 10atm에서 시작. 진공으로 자유팽창. $\boxed{w=0}$ ⇒ 온도그대로. 부피&압력변화있음

$T_1 = 300 \quad V_1 = 2.46(\ell) \qquad \longrightarrow \qquad T_2 = 300 \quad V_2 = 7.38(\ell)$

$P_1 = 10 \quad n_1 = 1 \qquad \qquad P_2 = \frac{10}{3} \quad n_2 = 1$

$q = 0, \quad \Delta U = 0 \quad \Delta H = \Delta U + \Delta(PV) = 0 \quad w = 0$

$\Delta S = nR\ln\frac{V_2}{V_1} = 8.314\ln 3 = 9.134(J/K)$

Q. 왜 $\Delta S = 0$ 이 아닌가요? — 자유팽창에서 온도가 균일하지 않아서가 맞을까요...??

b) reveribly heated 400K at constant Volume

$T_2 = 300 \quad V_2 = 7.38 \quad \longrightarrow \quad T_3 = 400 \quad V_3 = 7.38(\ell)$

$P_2 = \frac{10}{3} \quad n_2 = 1 \qquad\qquad P_3 = \frac{40}{9} \quad n_3 = 1$

$$P_3 = \frac{nRT}{V_3} = \frac{1\times0.082\times400}{7.38} = 4.44.. \left(\frac{40}{9}\right)(\ell)$$

$$\Delta S = \int \frac{\delta q}{T} = \int nC_v \cdot \frac{1}{T} dT = \frac{3}{2}\times 8.314 \ln\frac{4}{3} = 3.588\ (J/K)$$

$q = nC_v\Delta T = \frac{3}{2}\times8.314\times100 = 1247.1(J)$

$w = P\Delta V = 0$

$\Delta U = q - w = 1247.1(J)$

$\Delta H = \Delta U + \Delta(PV) = 1247.1 + 830 = 2078.0$

$$\boxed{\begin{aligned} q &= 1247.1(J) \\ w &= 0(J) \\ \Delta U &= 1247.1(J) \\ \Delta H &= 2078.0(J) \\ \Delta S &= 3.588(J/K) \end{aligned}}$$

c) 가역 & 등온, $V\times3$ $(P_3V_3 = P_4V_4)$

$T_3 = 400 \quad V_3 = 7.38 \quad \Longrightarrow \quad T_4 = 400 \quad V_4 = 22.14$

$P_3 = \frac{40}{9} \quad n_3 = 1 \qquad\qquad P_4 = \frac{40}{27} \quad n_4 = 1$

$$\Delta S = nR\ln\frac{V_4}{V_3} = 8.314\ln3 = 9.134(J/K)$$

등온 $\Rightarrow \Delta U = 0. \rightarrow \Delta U = q - w \Rightarrow q = 3653.5\ (J)$

$$w = \int PdV = nRT\ln\frac{V_4}{V_3} = 3653.5\ (J)$$

$\Delta H = \Delta U + \Delta(PV) = 0$

$$\boxed{\begin{aligned} q &= 3653.5\ (J) \\ w &= 3653.5(J) \\ \Delta U &= 0 \quad \Delta H = 0 \\ \Delta S &= 9.134(J/K) \end{aligned}}$$

d) reversibly cooled to 300K at constant pressure

$T_4 = 400 \quad V_4 = 22.14 \quad \Longrightarrow \quad T_5 = 300 \quad V_5 = 16.61$

$P_4 = \frac{40}{27} \quad n_4 = 1 \qquad\qquad P_5 = \frac{40}{27} \quad n_5 = 1$

· $dU = \delta q - \delta w \rightarrow \delta q = dU + \delta w$

$$\int\frac{\delta q}{T} = \int\frac{1}{T}dU + \int\frac{1}{T}\delta w$$

$$= \int\frac{nC_v dT}{T} + \int\frac{P}{T}dV = \int\frac{nC_v}{T}dT + nR\ln\frac{V_2}{V_1}$$

$$= nC_v\ln\frac{T_5}{T_4} + nR\ln\frac{V_5}{V_4}$$

$$= -3.588 + (-2.389) = -5.977(J/K)$$

$$\boxed{\begin{aligned} \Delta S &= -5.977J \\ w &= -830.1J \\ q &= -2078.5J \\ \Delta H &= -2078.5J \\ \Delta U &= -1248.4J \end{aligned}}$$

$w = P\Delta V = \frac{40}{27}\times(16.61 - 22.14) = -830.1\ (J)$

$q = nC_p\Delta T = \frac{5}{2}\times8.314\times(300-400) = -2078.5(J)$

$\Delta H = \Delta U + \Delta(PV) = -2078.5(J)$

3.

minimum...!!

3.(a) Find the extreme value of the function,

$$z = (x - 2)^2 + (y - 2)^2 + 4.$$

Find the constrained $\boxed{\text{maximum}}$ of this function corresponding to the condition

$$x + y = 1 \longrightarrow y = 1 - x$$

(b) by eliminating one variable and (c) by using a Lagrange undetermined multiplier method.

a) $f(x,y) = (x-2)^2 + (y-2)^2 + 4$

$f_x = 2(x-2) = 2x-4 \qquad f_y = 2(y-2) = 2y-4$

$\rightarrow x=2,\ y=2$일 때 extreme value $\boxed{z=4}$

$f_{xx} = 2,\ f_{yy} = 2,\ f_{xy} = 0 \Rightarrow D = 4 - 0 = 4 > 0$

$f_{xx} > 0$ $\Rightarrow$ minimum value이다.

b) $y = 1-x$

$z = (x-2)^2 + (-1-x)^2 + 4 = x^2 - 4x + 4 + x^2 + 2x + 1 + 4 = 2x^2 - 2x + 9$

$\frac{dz}{dx} = 4x - 2 = 0 \rightarrow x = \frac{1}{2}$일 때 $\boxed{z = \frac{17}{2}}$로 minimum.

c) by using Lagrange undetermined multiplier method.

$dz = 2(x-2)dx + 2(y-2)dy, \quad dy = -dx$

$= 2(x-2)dx - 2(y-2)dx$

$$\begin{cases} 2x - 4 = \lambda \\ 2y - 4 = \lambda \\ x + y = 1 \end{cases}$$

$2 - 2x - 4 = 2x - 4 \qquad x = \frac{1}{2},\ y = \frac{1}{2},\ \lambda = -3$

$\therefore x = \frac{1}{2},\ y = \frac{1}{2}$일 때 $\boxed{z = \frac{17}{2}}$

4. A rigid container is divided into two compartments of equal volume by a partition. One compartment contains 1 mole of ideal gas A at 1 atm, and the other compartment contains 1 mole of ideal gas B at 1 atm.

(a) Calculate the entropy increase in the container if the partition between the two compartments is removed.

(b) If the first compartment had contained 2 moles of ideal gas A, what would have been the entropy increase due to gas mixing when the partition was removed?

(c) Calculate the corresponding entropy changes in each of the above two situations if both compartments had contained ideal gas A.

a) A 1mol, P = 1atm. B 1mol, P=1atm.

i) A 입장: $\Delta S_A = R \ln \frac{V_2}{V_1} = R\ln 2$

ii) B 입장: $\Delta S_B = R\ln 2$

$\Rightarrow \Delta S = \boxed{2R\ln 2}$

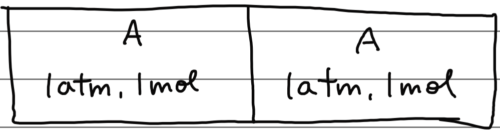
b) A 2mol, P=2atm, B 1mol, P=1atm

i) A 입장 : $\Delta S_A = 2R\ln\frac{V_B}{V_A} = 2R\ln 2$

ii) B 입장 : $R\ln 2$ $\Rightarrow$ $\Delta S = \boxed{3R\ln 2}$

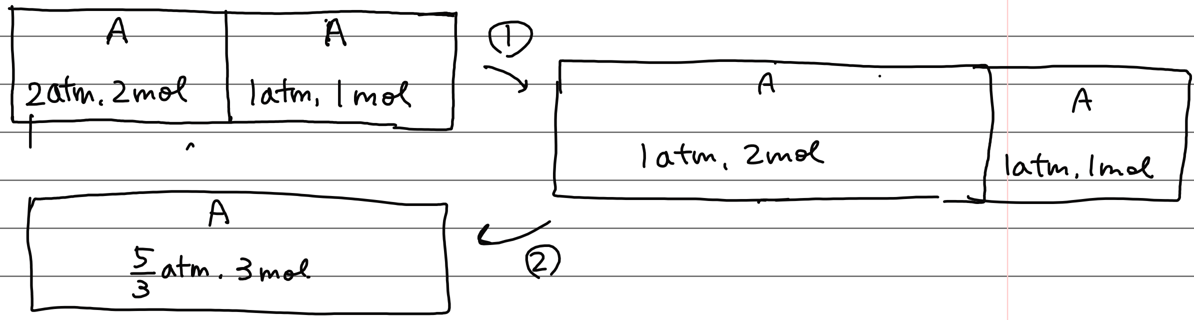
c) Calculate the corresponding entropy changes if both have A

a)



→ mix 이후도 그대로 → $\boxed{\Delta S = 0}$

b)



①: $\Delta S = 2R\ln 2$

②: $\Delta S = 3R\ln\frac{2}{3}$

$\Rightarrow \Delta S_{tot} = R\ln 4 + R\ln\frac{8}{27}$

$= \boxed{R\ln\frac{32}{27}}$