1. $C_{(x,t)} = (38 \text{ at\% Au}) + (12 \text{ at\% Au}) \exp[R(\beta)t] \cos\beta x$

(a) $\frac{\partial C}{\partial t} = R(\beta) \, (12 \text{ at\% Au}) \exp[R(\beta)t] \cos\beta x$

$\frac{\partial C}{\partial x} = -\beta \, (12 \text{ at\% Au}) \exp[R(\beta)t] \sin\beta x$

$\frac{\partial^2 C}{\partial x^2} = -\beta^2 (12 \text{ at\% Au}) \exp[R(\beta)t] \cos\beta x$

Fick's law $\Rightarrow \frac{\partial C}{\partial t} = \tilde{D} \frac{\partial^2 C}{\partial x^2}$

$R(\beta) \, \cancel{(12 \text{ at\% Au}) \exp[R(\beta)t] \cos\beta x} = \tilde{D} \times (-\beta^2) \cancel{(12 \text{ at\% Au}) \exp[R(\beta)t] \cos\beta x}$

$$R(\beta) = \tilde{D} \times (-\beta^2)$$

$\cos\beta x \sim [-1, 1]$

$C_{max}(x,t) - C_{min}(x,t) = (24 \text{ at\% Au}) \exp[R(\beta)t]$

$= (2 \text{ at\% Au})$

$\Rightarrow \exp[R(\beta)t] = \frac{1}{12}$

$\exp[\tilde{D} \times (-\beta^2) t] = \frac{1}{12} \quad \Rightarrow \quad t = 2.5 \times 10^4 \text{ s}$

$\tilde{D} = 10^{-23} \text{ m}^2/\text{s}, \ \beta = \frac{2\pi}{\lambda} \ (\lambda = 2 \times 10^{-9} \text{ m})$

(b) $$\frac{\partial c}{\partial t} = \tilde{D}\frac{\partial^2 c}{\partial x^2} - \frac{2K\tilde{D}}{f''}\frac{\partial^4 c}{\partial x^4}$$

$$\frac{\partial^3 c}{\partial x^3} = \beta^3 (12 at\%\ Au) \exp[R(\beta)t] \sin\beta x$$

$$\frac{\partial^4 c}{\partial x^4} = \beta^4 (12 at\%\ Au) \exp[R(\beta)t] \cos\beta x$$

$$R(\beta)\cancel{(12 at\%\ Au)}\cancel{\exp[R(\beta)t]\cos\beta x} = \tilde{D}\times(-\beta^2)\cancel{(12 at\%\ Au)}\cancel{\exp[R(\beta)t]\cos\beta x}$$
$$- \frac{2K\tilde{D}}{f''}\beta^4\cancel{(12 at\%\ Au)}\cancel{\exp[R(\beta)t]\cos\beta x}$$

$$\therefore R(\beta) = -\tilde{D}\beta^2 - \frac{2K\tilde{D}}{f''}\beta^4$$

$$C_{max} - C_{min} = (24 at\%\ Au)\exp[R(\beta)t] = (2 at\%\ Au)$$

$$\exp[R(\beta)t] = \frac{1}{12}$$

$$\exp\left(\left[-\tilde{D}\beta^2 - \frac{2K\tilde{D}}{f''}\beta^4\right]t\right) = \frac{1}{12} \Rightarrow t = 2.8\times10^4\ s$$

$\tilde{D} = 10^{-23}\ m^2/s$, $\beta = \frac{2\pi}{\lambda}$ $(\lambda = 2\times10^{-9}\ m)$, $f'' = 5\times10^{9}\ J/m^3$, $K = -2.6\times10^{-11}\ J/m$

(C) Cahn's modified diffusion equation에서는 $\frac{2K\tilde{D}}{f''}$ 라는 interfacial Energy에 관한 항이 있어, nonclassical nucleation과 같이 interfacial energy가 dominant한 경우에선 Fick's law와의 차이가 클 것이다.
Ag-Au system의 경우에는 ordering이 일어나 composition fluctuation의 영향이 적은 것이기에, nonclassical nucleation에 해당하지 않는다.
이에, $\frac{2K\tilde{D}}{f''}$ 항의 영향이 적어, (a) & (b)의 결과가 유사하게 나타난다.