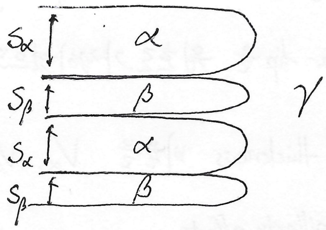
[AMSE 502] Phase Transformations H.W #6

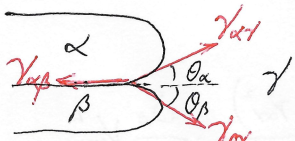
20202966 이정완

1. Formation of lamellar eutectic / eutectoid.



Free energy increase per mole $\Delta G_{IF}(S) = \dfrac{2\gamma_{\alpha\beta}}{S} \cdot V_m^L$

a) 계면에서 작용하는 힘의 평형을 보면,



$$\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\theta_\alpha + \gamma_{\beta\gamma}\cos\theta_\beta$$

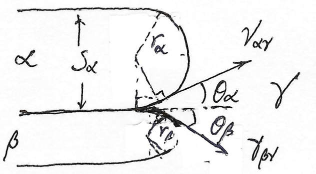
Curvature 가 없이 수직으로 각이 져 있다면 $\theta_\alpha = \theta_\beta = 90°$

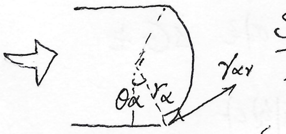
따라서, $\gamma_{\alpha\beta} = 0$ 이므로 성립할 수 없다.

$\therefore$ 양의 $\gamma_{\alpha\beta}$ 값을 가지기 위해 각각의 layer는 curvature 를 지니고 있다.

b) Radius of curvature as a function of thickness of layer.

각 layer의 tip 부분은 호의 형태로 되어 있다고 가정하자. α, β layer 호의 반지름 : r_α, r_β.





$$\frac{S_\alpha}{2} = r_\alpha\cos\theta_\alpha$$

$$\begin{cases} S_\alpha = 2r_\alpha\cos\theta_\alpha \longrightarrow \cos\theta_\alpha = \dfrac{S_\alpha}{2r_\alpha} \\ S_\beta = 2r_\beta\cos\theta_\beta \end{cases}$$

a) 에서와 같이 힘의 평형을 보면,

$$\begin{cases} \gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\theta_\alpha + \gamma_{\beta\gamma}\cos\theta_\beta \\ \gamma_{\alpha\gamma}\sin\theta_\alpha = \gamma_{\beta\gamma}\sin\theta_\beta \end{cases}$$

$$\gamma_{\alpha\gamma}^2(1-\cos^2\theta_\alpha) = \gamma_{\beta\gamma}^2(1-\cos^2\theta_\beta)$$

$$\cos^2\theta_\beta = 1 - \frac{\gamma_{\alpha\gamma}^2}{\gamma_{\beta\gamma}^2}(1-\cos^2\theta_\alpha) = \frac{\gamma_{\alpha\gamma}^2\cos^2\theta_\alpha - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2}{\gamma_{\beta\gamma}^2}$$

$$\cos\theta_\beta = \frac{\sqrt{\gamma_{\alpha\gamma}^2\cos^2\theta_\alpha - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2}}{\gamma_{\beta\gamma}} \quad (\theta_\beta < \tfrac{\pi}{2} \text{ 이므로 } \cos\theta_\beta \text{ 는 양수.})$$

$$\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\theta_\alpha + \sqrt{\gamma_{\alpha\gamma}^2\cos^2\theta_\alpha - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2}$$

$$= \frac{\gamma_{\alpha\gamma}S_\alpha}{2r_\alpha} + \sqrt{\frac{\gamma_{\alpha\gamma}^2 S_\alpha^2}{4r_\alpha^2} - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2}$$

$$\gamma_{\alpha\beta}^2 - \frac{\gamma_{\alpha\beta}\gamma_{\alpha\gamma}S_\alpha}{r_\alpha} + \cancel{\frac{\gamma_{\alpha\gamma}^2 S_\alpha^2}{4r_\alpha^2}} = \cancel{\frac{\gamma_{\alpha\gamma}^2 S_\alpha^2}{4r_\alpha^2}} - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2$$

$$\therefore r_\alpha = \frac{\gamma_{\alpha\beta}\gamma_{\alpha\gamma}S_\alpha}{\gamma_{\alpha\beta}^2 + \gamma_{\alpha\gamma}^2 - \gamma_{\beta\gamma}^2}$$

마찬가지로,

$$r_\beta = \frac{\gamma_{\alpha\beta}\gamma_{\beta\gamma}S_\beta}{\gamma_{\alpha\beta}^2 + \gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2}$$

$\therefore r_\alpha$ 와 r_β 는 각각 $f(S_\alpha)$, $f(S_\beta)$ 이다.

c) Free energy increase due to capillarity effects.

Generally, ΔG due to capillarity effects in this situation :

Layer tip을 원호로 가정하였으므로 $\Delta G = \frac{\gamma_{\alpha\gamma}}{r_\alpha} V_\alpha + \frac{\gamma_{\beta\gamma}}{r_\beta} V_\beta$

Layer thickness 비율을 V_α, V_β 비율과 같다고 보면 $V_\alpha = \frac{S_\alpha}{S} V_m^L$, $V_\beta = \frac{S_\beta}{S} V_m^L$.
$(S = S_\alpha + S_\beta)$

By capillarity effects,

$$\Delta G = \frac{\gamma_{\alpha\gamma}}{r_\alpha} \cdot \frac{S_\alpha}{S} V_m^L + \frac{\gamma_{\beta\gamma}}{r_\beta} \cdot \frac{S_\beta}{S} V_m^L$$

$$= \frac{\cancel{\gamma_{\alpha\gamma}}(\gamma_{\alpha\beta}^2 + \gamma_{\alpha\gamma}^2 - \gamma_{\beta\gamma}^2)}{\gamma_{\alpha\beta} \cancel{\gamma_{\alpha\gamma}} \cancel{S_\alpha}} \cdot \frac{\cancel{S_\alpha}}{S} V_m^L + \frac{\cancel{\gamma_{\beta\gamma}}(\gamma_{\alpha\beta}^2 + \gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2)}{\gamma_{\alpha\beta} \cancel{\gamma_{\beta\gamma}} \cancel{S_\beta}} \cdot \frac{\cancel{S_\beta}}{S} V_m^L$$

$$= \frac{2\gamma_{\alpha\beta}^2 V_m^L}{\gamma_{\alpha\beta} \cdot S}$$

$$= \frac{2\gamma_{\alpha\beta} V_m^L}{S}$$

$$= \Delta G_{IF}(S)$$

$\therefore$ Capillarity effects로 보인 α/β interface 생성에 따른 ΔG는 α/β interfacial energy로 얻은 ΔG와 일치한다.