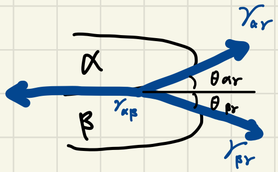
상변태론 HW 6.

1. a) For force balance,



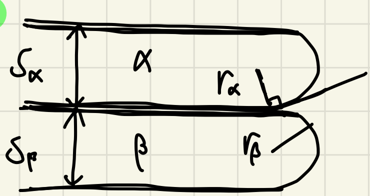
$$\gamma \begin{cases} \gamma_{\alpha\beta} = \gamma_{\alpha r}\cos\theta_{\alpha r} + \gamma_{\beta r}\cos\theta_{\beta r} \\ \gamma_{\alpha r}\sin\theta_{\alpha r} = \gamma_{\beta r}\sin\theta_{\beta r} \end{cases}$$

If there is no curvature, $\theta_{\alpha r} = \theta_{\beta r} = 90^\circ$, and $\gamma_{\alpha\beta}$ should be 0.

$\longrightarrow$ Impossible.

So, the layer tip should has curvature.

b)



$$S_\alpha = 2r_\alpha\cos\theta_{\alpha r}$$
$$S_\beta = 2r_\beta\cos\theta_{\beta r}$$

$$r_{\alpha r}\sin\theta_{\alpha r} = r_{\beta r}\sin\theta_{\beta r}$$
$$r_{\alpha r}^2\sin^2\theta_{\alpha r} = r_{\beta r}^2\sin^2\theta_{\beta r}$$
$$r_{\alpha r}^2(1-\cos^2\theta_{\alpha r}) = r_{\beta r}^2(1-\cos^2\theta_{\beta r})$$

$$\cos\theta_{\beta r} = \sqrt{\frac{r_{\beta r}^2 - \gamma_{\alpha r}^2 + r_{\alpha r}^2\cos^2\theta_{\alpha r}}{\gamma_{\beta r}^2}}$$

$$\gamma_{\alpha\beta}{}^2 = \gamma_{\alpha r}\cos\theta_{\alpha r} + \gamma_{\beta r}\sqrt{\frac{\gamma_{\beta r}^2 - \gamma_{\alpha r}^2 + \gamma_{\alpha r}^2\cos^2\theta_{\alpha r}}{\gamma_{\beta r}^2}}$$

$$= \gamma_{\alpha r}\frac{S_\alpha}{2\gamma_\alpha} + \sqrt{r_{\beta r}^2 - \gamma_{\alpha r}^2 + \gamma_{\alpha r}^2\frac{S_\alpha^2}{4r_\alpha^2}}$$

$$r_{\beta r}^2 - r_{\alpha r}^2 + r_{\alpha r}^2\frac{S_\alpha^2}{4r_\alpha^2} = \gamma_{\alpha\beta}^2 - 2\gamma_{\alpha\beta}\gamma_{\alpha r}\cdot\frac{S_\alpha}{2\gamma_\alpha} + \gamma_{\alpha r}^2\cdot\frac{S_\alpha^2}{4r_\alpha^2}$$

$$\frac{1}{r_\alpha} = \frac{\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2}{S_\alpha \cdot \gamma_{\alpha\beta} \cdot \gamma_{\alpha\gamma}}$$

$$r_\alpha = \frac{S_\alpha \gamma_{\alpha\beta} \gamma_{\alpha\gamma}}{\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2} \qquad r_\beta = \frac{S_\beta \gamma_{\alpha\beta} \gamma_{\beta\gamma}}{\gamma_{\alpha\beta}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2}$$

c)

$$\begin{cases} \Delta G_{T\alpha} = \dfrac{\gamma_{\alpha\gamma} \cdot V_{\alpha\gamma}}{r_\alpha} \\ \Delta G_{T\beta} = \dfrac{\gamma_{\beta\gamma} \cdot V_{\beta\gamma}}{r_\beta} \end{cases}$$

$$\Delta G_T = \Delta G_{T\alpha} + \Delta G_{T\beta}$$

$$= \frac{\gamma_{\alpha\gamma} \cdot V_{\alpha\gamma}}{r_\alpha} + \frac{\gamma_{\beta\gamma} \cdot V_{\beta\gamma}}{r_\beta}$$

$$= \frac{(\gamma_{\alpha\beta}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2)\gamma_{\alpha\gamma} V_{\alpha\gamma}}{S_\alpha \gamma_{\alpha\beta} \gamma_{\alpha\gamma}} + \frac{(\gamma_{\alpha\beta}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\alpha\gamma}^2)\gamma_{\beta\gamma} V_{\beta\gamma}}{S_\beta \gamma_{\alpha\beta} \gamma_{\beta\gamma}}$$

Assume,

$$\frac{V_{\alpha\gamma}}{S_\alpha} = \frac{V_{\beta\gamma}}{S_\beta} = \frac{V_m}{S} \therefore \qquad \Delta G_T = \frac{2\gamma_{\alpha\beta} V_m}{S}$$