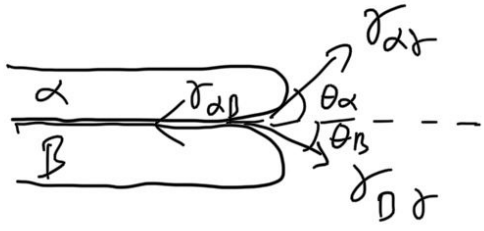
a) curvature 의 평상을 tip이 갈는 이유



Force balance equation

$$\Rightarrow \begin{cases} \gamma_{\alpha\beta} - \gamma_{\alpha\gamma}\cos\theta_\alpha - \gamma_{\beta\gamma}\cos\theta_\beta = 0 \quad (1) \\ \gamma_{\alpha\gamma}\sin\theta_\alpha - \gamma_{\beta\gamma}\sin\theta_\beta = 0 \quad (2) \end{cases}$$

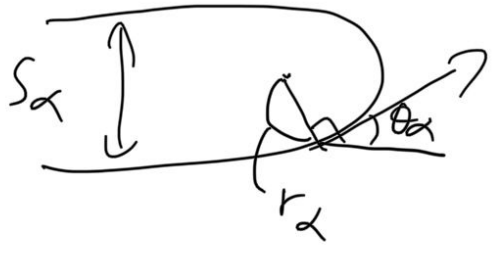
curvature가 없다면, $\theta_\alpha = \theta_\beta = 90^\circ$

$\Rightarrow \boxed{\gamma_{\alpha\beta} = 0}$ 인 경우 이다, 또는 $\gamma_{\alpha\gamma} = \gamma_{\beta\gamma}$ 인데

갈은 재료가 아닌 이상, 위 조건을 만족하기 어려우므로

항상 curvature 존재

(b)



$$\Rightarrow r_\alpha = \frac{S_\alpha}{2\cos\theta_\alpha}$$

$$\Rightarrow r_\beta = \frac{S_\beta}{2\cos\theta_\beta}$$

이를 (2) 식에 넣어 대입

$$\Rightarrow \cos\theta_B = \sqrt{1-\left(\frac{r_{\alpha\gamma}}{r_{B\gamma}}\right)^2(1-\cos^2\theta_\alpha)}$$

이를 (1) 에 대입

$$r_{\alpha B} = r_{\alpha\gamma}\cos\theta_\alpha + r_{B\gamma}\sqrt{1-\left(\frac{r_{\alpha\gamma}}{r_{B\gamma}}\right)^2(1-\cos^2\theta_\alpha)}$$

$$\Rightarrow \cos\theta_\alpha = \frac{r_{\alpha B}^2 - r_{B\gamma}^2 + r_{\alpha\gamma}^2}{2r_{\alpha B}\, r_{\alpha\gamma}}$$

$$\cos\theta_B = \frac{r_{\alpha B}^2 - r_{\alpha\gamma}^2 + r_{B\gamma}^2}{2r_{\alpha B}\, r_{B\gamma}}$$

$$\cos\theta_B = \frac{\gamma_{\alpha B}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{B\gamma}^2}{2\gamma_{\alpha B}\gamma_{B\gamma}}$$

$$r_\alpha = \frac{S_\alpha \gamma_{\alpha B}\gamma_{\alpha\gamma}}{\gamma_{\alpha B}^2 - \gamma_{B\gamma}^2 + \gamma_{\alpha\gamma}^2} \qquad r_B = \frac{S_B \gamma_{\alpha B}\gamma_{B\gamma}}{\gamma_{\alpha B}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{B\gamma}^2}$$

(C) Gibbs free E (capillary effect)

$$\Delta G_\alpha = \frac{\gamma_{\alpha\gamma} V_{\alpha\gamma}}{r_\alpha} \qquad \Delta G_B = \frac{\gamma_{B\gamma} V_{B\gamma}}{r_B}$$

$$\Delta G = \Delta G_\alpha + \Delta G_B = \frac{(\gamma^2_{\alpha B} - \gamma^2_{\alpha\gamma} + \gamma^2_{B\gamma})\gamma_{\alpha\gamma} V_\alpha}{S_\alpha \gamma_{\alpha B}\gamma_{\alpha\gamma}} + \frac{(\gamma^2_{\alpha B} - \gamma^2_{B\gamma} + \gamma^2_{\alpha\gamma})\gamma_{B\gamma} V_B}{S_B \gamma_{\alpha B}\gamma_{B\gamma}}$$

$$\Rightarrow \quad \Delta G = \frac{2\gamma_{\alpha_B} V_m}{S}$$